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FAULT SLIP AND SEISMICITY INDUCED BY SUBSURFACE FLUID WITHDRAWAL

FAULT 99

Report to Shell Exploration and Production Company

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TABLE OF CONTENTS

TABLE OF CONTENTS

Fault 99

1. Introduction

Schematics of general geologic setup is shown in Figure 1.1. Code, Fault 99 is a Mathcad 8 Pro file to analyze stresses and displacements around a depleted poroelastic reservoir as well as the stability of the nearby faults. It allows to compare different depletion/injection strategies and how they affect the fault stability. Fault 99 also allows to estimate of the magnitude of the fault slip due to the reservoir depletion or fluid injection in the reservoir. Fault 99 does account for the interaction between the slipping fault and depleting reservoir although the back influence of the reactivated part of the fault on the depleting reservoir is not included. This important option will be available in the next release of Fault 99.

The employed concept is based on the following assumptions:

(1) The reservoir is represented by a Winkler-like, poroelastic inclusion embedded in an elastic material.
(2) The material of the fault gouge obeys Mohr-Coulomb strength criterion.
(3) Reservoir is *deep*, *thin*, *smooth*, *soft*, and *isolated* (no interaction with the earth surface, other reservoirs and preexisting faults).
(4) The residual stresses are assumed to be known. Residual stresses are stresses that would remain in the absence of remote stresses and reservoir pore pressure.
(5) Two-dimensional (plain strain) conditions are considered.

The modular structure of the employed algorithm allows the user to easily change material parameters, reservoir shape, pressure distribution, and fault slip criterion. The user has to be only minimally familiar with Mathcad 8 Pro to be able to use Fault 99.

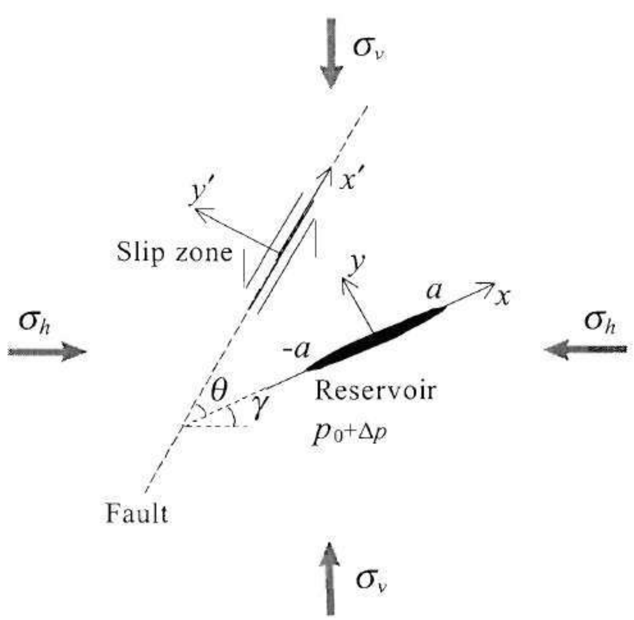


Figure 1.1. Fault and reservoir positions.

1.1. Main Parameters, Definitions, and Notations

NOTE: The highlighted expressions below represent important places where the introduced parameters and functions are defined (specified).

$MPa \equiv 10^6 \cdot Pa$

$E := 1 \cdot 10^{10} \cdot Pa$	Young's modulus of surrounding rock
$E_1 := \frac{1}{3} \cdot E$	Young's modulus of reservoir rock
$\nu := 0.25$	Poisson's ratio of surrounding rock
$\nu_1 := 0.35$	Poisson's ratio of reservoir rock
$\lambda_1(E_1, \nu_1) := \frac{E_1 \cdot \nu_1}{(1+\nu_1) \cdot (1 - 2 \cdot \nu_1)}$	Lame parameter of reservoir material
$\mu(E, \nu) := \frac{E}{2 \cdot (1+\nu)}$	Shear modulus of surrounding material
$\mu_1(E_1, \nu_1) := \frac{E_1}{2 \cdot (1+\nu_1)}$	Shear modulus of reservoir material
$a := 150 \cdot m$	Half-length of depleted part of reservoir
$h_{max} := 30 \cdot m$	Maximum thickness of reservoir
$h(h_{max}, f_h, x) := h_{max} \cdot f_h(x)$	Reservoir thickness
$f_h(x) := 1$ $\quad f_h(x) \le 1$	Dimensionless distribution of reservoir thickness, i.e., $f_h(x) = \frac{h(x)}{a}$
	IMPORTANT! **Function $f_h(x)$ has to be redefined every time the shape of the reservoir is changed.**
$\rho_r \equiv 2500 \cdot \frac{kg}{m^3}$	Rock density
$g = 9.8066\ m \cdot s^{-2}$	Acceleration due to gravity
$H \equiv 14000 \cdot ft$	Depth of reservoir (needed only for in-situ stress estimate)
$\sigma_1 \equiv -\rho_r \cdot g \cdot H$	Major principal stress

$\sigma_1 := -100 \cdot MPa$

NOTE: Type in the σ_1 value directly if you do not want to use the lithostatic pressure, i.e., weight based estimate. Otherwise, disable the second expression.

$\lambda := 0.75$ In-situ stress ratio, i.e., $\lambda = \sigma_3/\sigma_1$

$\sigma_3(\lambda, \sigma_1) := \lambda \cdot \sigma_1$ Minor principal stress

NOTE: Major and minor principal stresses are the greatest and least stress components in terms of their absolute value since according to the adopted sign convention, compressive stresses are negative. These stresses do not necessarily coincide with σ_v and σ_h shown in Figure 1.1. For example, in the reversed/thrust stress regime, the major principal stress is horizontal while the vertical stress is minor. In Fault 99, notations, σ_1 and σ_3, do not necessarily correspond to major and minor principal stresses.

$\theta := 60 \cdot deg$ Angle between the fault and reservoir

$\gamma := 0 \cdot deg$ Angle between the reservoir and minor principal stress

$\psi(\gamma, \theta) := \theta + \gamma$ Angle between the fault and minor principal stress

ϕ Angle of internal friction of fault gouge material

$\kappa = \tan(\phi)$ Coulomb friction coefficient

C Cohesion of fault gouge material

$\alpha_p := 1$ Biot's poroelastic constant

$N := 100$ Number of collocation points

IMPORTANT!
The number of collocation points, *N*, is specified for the whole file.

$p_0 := 100 \cdot MPa$ Reservoir pressure before depletion/injection

NOTE: From the physical stand point, p_0 should not be too high to avoid the possibility of hydraulic fracturing of the reservoir and/or the surrounding material. Therefore, the user is recommended to check the stresses both inside and outside the reservoir calculated in Sections 10.3 and 10.4. It is generally safe to make sure that all the normal stress components are compressive (negative) everywhere.

$\Delta p_{max} := 0 \cdot Pa$ Maximum pressure depletion/injection in the reservoir (will be redefined later as needed)

$\Delta p_{max\_0} := 0 \cdot Pa$ *Initial* maximum pressure depletion/injection in the reservoir (will not be redefined anymore, needed to compute initial or residual stress distributions)

$f_{\Delta p}(\xi) := 1$ $\quad \left| f_{\Delta p}(\xi) \right| \le 1$ Dimensionless distribution of pressure depletion/injection in reservoir

IMPORTANT!
Function $f_{\Delta p}(x)$ has to be redefined every time the distribution of pressure depletion/injection in the reservoir is changed.

$\Delta p(\Delta p_{max}, f_{\Delta p}, \xi) := \Delta p_{max} \cdot f_{\Delta p}(\xi)$ Distribution of pressure perturbation (depletion/injection) in the reservoir

$p(\xi) = p_0 + \Delta p(\xi)$ Current pressure in the depleted reservoir

$p_f := 30 \cdot MPa$ Pore pressure in the fault gouge

$\xi = \frac{x}{a}$ Dimensionless coordinate along the reservoir

$\eta = \frac{y}{a}$ Dimensionless coordinate normal to the reservoir

$z = x + i \cdot y$ Complex variable associated with the reservoir coordinate set (Figure 1.1)

$\zeta = \frac{z}{a}$ The same as above but normalized

$\eta_f = \frac{h_f}{a}$ Point where the fault intersects the η axis

ξ_f Point where the fault intersects the ξ axis

IMPORTANT!
Sign convention: compression is negative, pore pressure is positive, depletion corresponds to $\Delta p<0$. Fluid injection corresponds to $\Delta p>0$.

$\sigma_0 := \left| \sigma_1 \right|$ Parameter used for normalizing stresses (can be any nonzero quantity of stress dimension)

IMPORTANT!
Because in many places σ_0 is present in the denominator, it is the user responsibility to make sure that its value is not zero. For example, if $\sigma_1=0$, its value cannot be used for normalizing.

$z' = x' + i \cdot y'$ Complex variable associated with the fault (Figure 1.1)

$\zeta' = \frac{z'}{a}$ Same as above but normalized

1.2. Mathematical Model

In a usual way, the problem under consideration is decomposed into two problems (Figure 1.2):

(1) *auxiliary problem* for the reservoir (zero remote in-situ stresses) and

(2) *reference state problem* with remote and residual stresses without the reservoir.

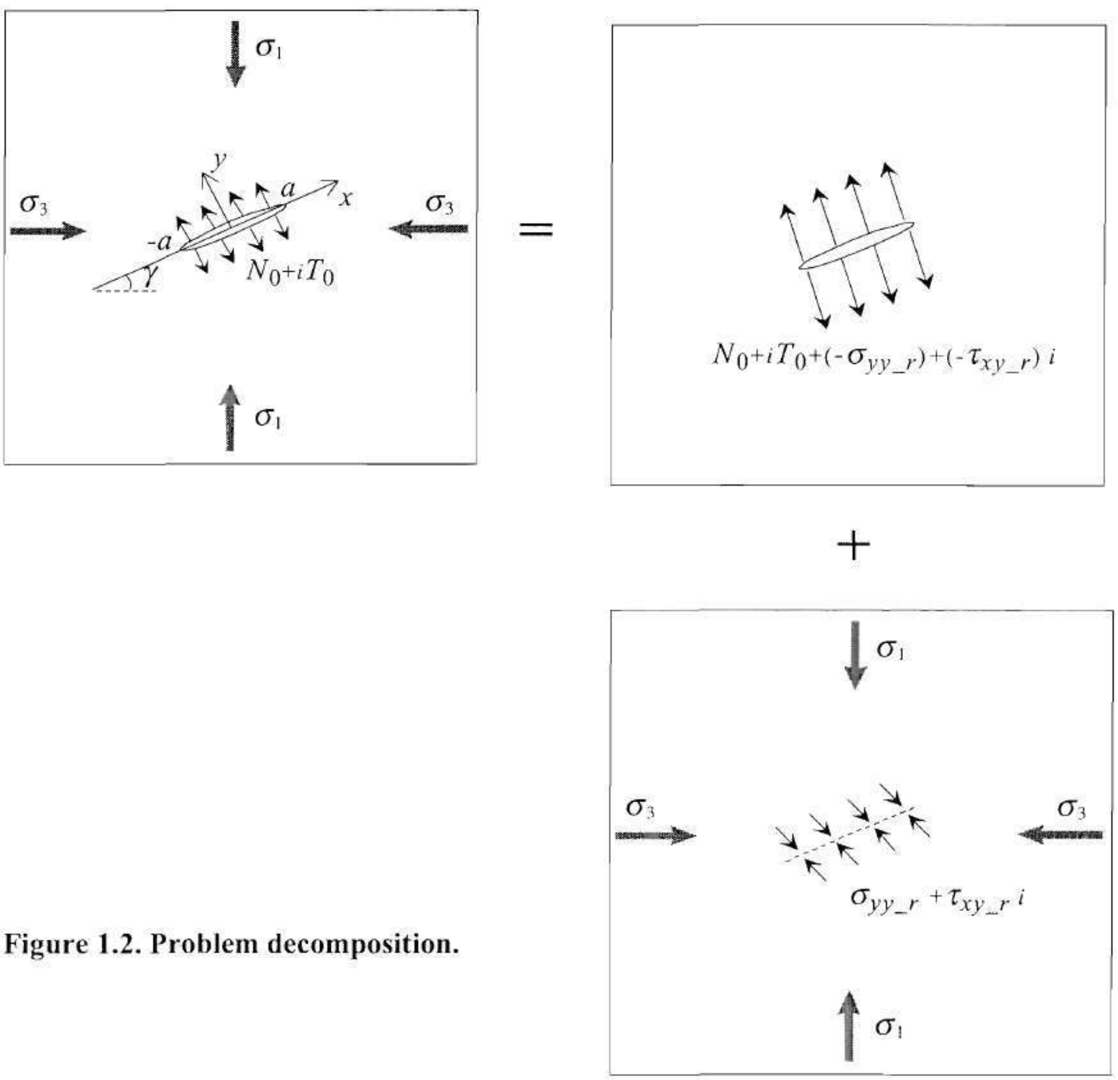
NOTE: In the *original problem* shown in Figure 1.2, the residual stresses are not shown but present implicitly. Accordingly, they are also implicitly present in the *reference state problem*. Therefore, reference stresses below include both the remote and residual stresses defined by Fault 99 (see below).

Original Problem

Reservoir pressure $p(x)$

Auxiliary Problem

Reservoir pressure $p(x)$



Reference State Problem

No inclusion

Figure 1.2. Problem decomposition.

Residual stresses are those that would exist even in the absence of the remote stresses, σ_v and σ_h, and the reservoir pore pressure, p (Figure 1.2). As the reservoir depletes, the total stresses are perturbed but the residual stresses are not affected. It is this stress perturbation, which, at least in principle, can be modeled if the materials involved are linear. However, fault stability is affected not by the stress perturbations but rather by the total shear stress on the fault. At any stage of depletion, the total stresses are the sum of the initial in-situ stresses and the stress perturbations. Therefore, knowing the initial in-situ stresses is essential for modeling.

It is usually recognized that accurate evaluation of all the components of the in-situ stresses may not be not feasible. They are affected by various factors including the presence of the pressurized reservoir disturbing the stress field inside and around it. Nevertheless, a reasonable distribution of either the initial in-situ stresses or the residual stresses has to be assumed to make the model fully determined. In Fault 99, we have provided two extreme options for residual stresses:

1. Zero residual stress so that the initial in-situ stresses are completely defined by the remote stresses and the reservoir pressure.
2. Residual stresses are such that the total stresses are lithostatic before depletion.

The residual stresses are constant for a given set of parameters and do not change when the pressure is changing (see Section 4.4).

The logic of the solution, described by Germanovich et al. (1999), is as follows:

1. Find the relation between displacements of reservoir sides and tractions generated by the reservoir deformation in the approximation of *generalized* Winkler's conditions. We have for the *original problem* (Figure 1.2):

$$N_0(x) = k(x)\cdot\Delta v_0(x) - \frac{\lambda_1}{2\cdot h(x)}\cdot\frac{d}{dx}\left(h_{top}(x)+h_{bot}(x)\right)\cdot\Delta u_0(x) + \lambda_1\cdot\frac{d}{dx}u_{0\_av}(x) - \alpha_p\cdot p(x) \qquad (1.1a)$$

$$u_{0\_av}(x) = \frac{u_0\left(x+i\cdot h_{top}\right) + u_0\left(x-i\cdot h_{bot}\right)}{2} \qquad (1.1b)$$

$$T_0(x) = m(x)\cdot\Delta u_0(x) - \frac{\mu_1}{2\cdot h(x)}\cdot\frac{d}{dx}\left(h_{top}(x)+h_{bot}(x)\right)\cdot\Delta v_0(x) + \mu_1\cdot\frac{d}{dx}v_{0\_av}(x) \qquad (1.1c)$$

$$v_{0\_av}(x) = \frac{v_0\left(x+i\cdot h_{top}\right) + v_0\left(x-i\cdot h_{bot}\right)}{2} \qquad (1.1d)$$

where $p(x) = p_0 + \Delta p(x)$ is the current reservoir pressure, $k(x)$ and $m(x)$ are normal and shear reservoir rigidities, respectively,

$$k(x) = \frac{\lambda_1 + 2\cdot\mu_1}{h(x)} \qquad m(x) = \frac{\mu_1}{h(x)} \qquad (1.2)$$

and $\Delta v_0(x) + i \cdot \Delta u_0(x)$ are the displacement discontinuities between the upper and lower interfaces of the reservoir in the *original problem* under consideration (Figure 1.2), and u_{0_av} and v_{0_av} are the displacements averaged over y inside the reservoir (it is assumed that the displacements vary linearly across the thickness of the thin reservoir). For a symmetric reservoir (with respect to x-axis), the second terms in (1.1) are identically zero. It is further assumed that the reservoir is symmetrical with respect to x-axis.

NOTE: If the second terms in (1.1) are retained (in case of asymmetrical reservoir), then equations for the calculation of the coefficients, α's and β's, will be coupled. However, the adopted approach can still be implemented although in the case of the symmetrical reservoir, the calculations are much simpler because equations for α's and β's become independent of each other.

2. Find remote in-situ stresses in the reservoir coordinate set (x, y) shown in Figures 1.1 and 1.2 (*remote stress problem*):

$$\sigma_{yy\_remote}(\gamma,\lambda,\sigma_1) := \sigma_1 \cdot \cos(\gamma)^2 + \sigma_3(\lambda,\sigma_1) \cdot \sin(\gamma)^2 \tag{1.3}$$

$$\sigma_{xx\_remote}(\gamma,\lambda,\sigma_1) := \sigma_1 \cdot \sin(\gamma)^2 + \sigma_3(\lambda,\sigma_1) \cdot \cos(\gamma)^2 \tag{1.4}$$

$$\tau_{xy\_remote}(\gamma,\lambda,\sigma_1) := \frac{\sigma_1 - \sigma_3(\lambda,\sigma_1)}{2} \cdot \sin(2\cdot\gamma) \tag{1.5}$$

Total normal stress (which may be used as a reference for stress normalization) is:

$$\sigma_N(\gamma,\lambda,\sigma_1,\Delta p_{max},p_0,\alpha_p) := \sigma_{yy\_remote}(\gamma,\lambda,\sigma_1) + \alpha_p \cdot (\Delta p_{max} + p_0) \tag{1.6}$$

3. Introduce tractions on the reservoir surface in the *auxiliary problem* (Figure 1.2):

$$N(x) = N_0(x) - \sigma_{yy\_remote} \tag{1.7}$$

$$T(x) = T_0(x) - \tau_{xy\_remote} \tag{1.8}$$

The displacements and displacement discontinuities in the *original problem* can be represented as:

$$u_0 = u + u_{remote} \qquad v_0 = v + v_{remote} \qquad \Delta u_0 = \Delta u + \Delta u_{remote} \qquad \Delta v_0 = \Delta v + \Delta v_{remote} \tag{1.9}$$

where $\Delta v(x) + i \cdot \Delta u(x)$ are the displacement discontinuities between the upper and lower interfaces of the reservoir in the *auxiliary problem*.

4. Find tractions on the crack sides in the *auxiliary problem:*

$$N(x) = k(x)\cdot\Delta v(x) + \lambda_1 \frac{d}{dx} u_{av}(x) + (\lambda_1 + 2\cdot\mu_1)\cdot\varepsilon_{yy\_remote} + \lambda_1 \cdot \varepsilon_{xx\_remote} - \alpha_p \cdot p(x) - \sigma_{yy\_remote} \tag{1.10}$$

$$u_{av}(x) = \frac{u(x + i \cdot 0) + u(x - i \cdot 0)}{2} \tag{1.10a}$$

$$T(x) = m(x)\cdot\Delta u(x) + \mu_1 \frac{d}{dx} v_{av}(x) + 2\cdot\mu_1\cdot\varepsilon_{xy\_remote} - \tau_{xy\_remote} \tag{1.11}$$

$$v_{av}(x) = \frac{v(x + i \cdot 0) + v(x - i \cdot 0)}{2} \tag{1.11a}$$

where

$$\varepsilon_{xx\_remote} = \frac{1 - v^2}{E}\cdot\left(\sigma_{xx\_remote} - \frac{v}{1 - v}\cdot\sigma_{yy\_remote}\right), \tag{1.12a}$$

$$\varepsilon_{yy\_remote} = \frac{1 - v^2}{E}\cdot\left(\sigma_{yy\_remote} - \frac{v}{1 - v}\cdot\sigma_{xx\_remote}\right) \tag{1.12b}$$

are the strains in the *x* and *y* directions, respectively, in the *remote stress problem*.

5. **Assume or obtain residual stress distribution.**

NOTE: In Fault 99, we have provided two options for residual stresses. First, zero residual stress and second, residual stresses are such that the total stresses are lithostatic before depletion. The residual stresses are constant for a given set of parameters and identified in Section 4.4.

$$\sigma_{yy\_r} = \sigma_{yy\_remote} + \sigma_{yy\_residual} \tag{1.13a}$$

$$\sigma_{xx\_r} = \sigma_{xx\_remote} + \sigma_{xx\_residual} \tag{1.13b}$$

$$\tau_{xy\_r} = \tau_{xy\_remote} + \tau_{xy\_residual} \tag{1.13c}$$

Identifying residual and remote stresses corresponds to choosing the *reference state* with respect to which all stresses and displacements are calculated. It would be convenient to suppose that the reference state corresponds to zero displacements, stresses, and reservoir pressure. However, because the remote stresses are *homogeneous* in Fault 99, this assumption does not correspond to the *lithostatic* conditions (when the initial stresses are also uniform throughout the formation). In other words, only one of initial and residual stresses can be assumed homogeneous but not both.

6. **Obtain the solution of the *original problem* (Figure 1.2) under consideration by solving the *auxiliary problem* and adding the stresses and displacements from the *reference state*.**

2. Numerical Algorithm

Numerical algorithm is based on the analytical formulae that generalize the McCartney and Gorley (1987) approach to the case of thin poroelastic inclusions. The algorithm is based on the formation of two matrices for the computation of coefficients α'sand β's. Then, stresses and displacements around the reservoir are calculated based on these coefficients through the derived analytical formulae.

2.1. McCartney and Gorley's (1987) Representation

McCartney and Gorley (1987) suggested a convenient representation of displacements and stresses in a plane with a crack. Their equations (34), (35), and (36) are reproduced below:

$$u - i \cdot v = \frac{a \cdot e^{i \cdot \theta}}{4 \cdot \mu} \cdot \left[\sum_{n=1}^{N} \frac{1}{n} \cdot \left(\alpha_n + i \cdot \beta_n \right) \cdot \left[G_n(\zeta) - \kappa \cdot \overline{\left[G_n(\zeta) \right]} \right] + \left(\zeta - \overline{\zeta} \right) \cdot \sum_{n=1}^{N} \left(\alpha_n - i \cdot \beta_n \right) \cdot \frac{G_n(\zeta)}{\sqrt{\zeta^2 - 1}} \right] \tag{2.1}$$

$$\frac{1}{2} \cdot \left(\sigma_{xx} + \sigma_{yy} \right) = \mathrm{Re} \left[\sum_{n=1}^{N} \left(\alpha_n - i \cdot \beta_n \right) \cdot \frac{G_n(\zeta)}{\sqrt{\zeta^2 - 1}} \right] \tag{2.2}$$

$$\frac{1}{2} \cdot \left(\sigma_{yy} - \sigma_{xx} \right) + i \cdot \tau_{xy} = i \cdot e^{2 \cdot i \cdot \theta} \cdot \sum_{n=1}^{N} \beta_n \cdot \frac{G_n(\zeta)}{\sqrt{\zeta^2 - 1}} \ldots \tag{2.3}$$

$$+ \frac{1}{2} \cdot e^{2 \cdot i \cdot \theta} \cdot \left(\zeta - \overline{\zeta} \right) \cdot \sum_{n=1}^{N} \left(\alpha_n - i \cdot \beta_n \right) \cdot \left(\frac{n}{\sqrt{\zeta^2 - 1}} + \frac{\zeta}{\zeta^2 - 1} \right) \cdot \frac{G_n(\zeta)}{\sqrt{\zeta^2 - 1}}$$

Displacement discontinuities between the reservoir sides are given by equation (27) in McCartney and Gorley (1987):

$$\Delta v(\xi) + i \cdot \Delta u(\xi) = \frac{4 \cdot a}{E'} \cdot \sqrt{1 - \xi^2} \cdot \sum_{n=1}^{N} \frac{1}{n} \cdot \left(\alpha_n + i \cdot \beta_n \right) \cdot U_{n-1}(\xi) \tag{2.4}$$

where $E' = \frac{E}{1 - v^2}$ for plane strain, $E' = E$ for plane stress, $U_n(\zeta)$ are the Chebyshev polynomia of the second kind, i.e., for $0 \le \alpha \le \pi$ $(n \ge 0)$,

$$U_n(\cos(\alpha)) = \frac{\sin((n+1) \cdot \alpha)}{\sin(\alpha)} \tag{2.4a}$$

Here $\zeta = \frac{z}{a} = \xi + i \cdot \eta$ is the normalized local coordinate system attached to the crack, θ is the the crack inclination to the *x*-axis in the global coordinate system (McCartney and Gorley's (1987) notations). Accordingly, in our case (Figure 1.1) the local and global coordinate sets of a discontinuity coincide, while θ is simply used to denote the angle between the fault and reservoir (Figure 1.1).

Adding Eqs. (2.2) and (2.3), we obtain:

$$\sigma_{yy}=\mathrm{Re}\left[\sum_{n=1}^{N}\left(\alpha_n-i\cdot\beta_n\right)\cdot\frac{G_n(\zeta)}{\sqrt{\zeta^2-1}}+i\cdot e^{2\cdot i\cdot\theta}\cdot\sum_{n=1}^{N}\beta_n\cdot\frac{G_n(\zeta)}{\sqrt{\zeta^2-1}}\ \ldots\right.$$
$$\left.+\frac{1}{2}\cdot e^{2\cdot i\cdot\theta}\cdot\left(\zeta-\overline{\zeta}\right)\cdot\sum_{n=1}^{N}\left(\alpha_n-i\cdot\beta_n\right)\cdot\left(\frac{n}{\sqrt{\zeta^2-1}}+\frac{\zeta}{\zeta^2-1}\right)\cdot\frac{G_n(\zeta)}{\sqrt{\zeta^2-1}}\right] \tag{2.5}$$

$$\tau_{xy}=\mathrm{Im}\left[i\cdot e^{2\cdot i\cdot\theta}\cdot\sum_{n=1}^{N}\beta_n\cdot\frac{G_n(\zeta)}{\sqrt{\zeta^2-1}}+\frac{1}{2}\cdot e^{2\cdot i\cdot\theta}\cdot\left(\zeta-\overline{\zeta}\right)\cdot\sum_{n=1}^{N}\left(\alpha_n-i\cdot\beta_n\right)\cdot\left(\frac{n}{\sqrt{\zeta^2-1}}+\frac{\zeta}{\zeta^2-1}\right)\cdot\frac{G_n(\zeta)}{\sqrt{\zeta^2-1}}\right] \tag{2.6}$$

$$\sigma_{xx}=2\cdot\mathrm{Re}\left[\sum_{n=1}^{N}\left(\alpha_n-i\cdot\beta_n\right)\cdot\frac{G_n(\zeta)}{\sqrt{\zeta^2-1}}\right]-\sigma_{yy} \tag{2.7}$$

where auxiliary functions are defined by

$$G_n(\zeta)=\left(\zeta-\sqrt{\zeta^2-1}\right)^n \qquad n\ge 1 \tag{2.8a}$$

$$g_n(\zeta)=I_n(\zeta)+i\cdot J_n(\zeta)=\frac{G_n(\zeta)}{\sqrt{\zeta^2-1}}=\frac{\left(\zeta-\sqrt{\zeta^2-1}\right)^n}{\sqrt{\zeta^2-1}} \tag{2.8b}$$

$$I(\zeta,n)=\mathrm{Re}\left[\frac{\left(\zeta-\sqrt{\zeta^2-1}\right)^n}{\sqrt{\zeta^2-1}}\right] \qquad J(\zeta,n)=\mathrm{Im}\left[\frac{\left(\zeta-\sqrt{\zeta^2-1}\right)^n}{\sqrt{\zeta^2-1}}\right] \tag{2.8c}$$

Here $\zeta=\xi+i\cdot\eta$ and $\zeta'=\xi'+i\cdot\eta'$ are the normalized coordinates associated with the reservoir and fault, respectively (Figure 1.1).

Taking into account that $G_n(\xi)=G_n(\xi+i\cdot 0)=\left(\xi-i\cdot\sqrt{1-\xi^2}\right)^n$ $(|\xi|<1)$, the average displacements inside the reservoir $(\zeta-\overline{\zeta}=2\cdot i\cdot\eta=0)$ can be written as

$$u_{av}-i\cdot v_{av}=\frac{a}{4\cdot\mu}\cdot(1-\kappa)\cdot e^{i\cdot\theta}\cdot\sum_{n=1}^{N}\frac{\alpha_n+i\cdot\beta_n}{n}\cdot\mathrm{Re}\left(\xi-i\cdot\sqrt{1-\xi^2}\right)^n \qquad \eta=0,\ |\xi|<1 \tag{2.9}$$

2.2. System of Linear Algebraic Equations

As demonstrated below (see also Germanovich et al., 1999), McCartney and Gorley's (1987) representation allows to conveniently solve problems not only for cracks but also for elastic inclusions. Generalizing their approach, we arrive at the following formulation.

Boundary condition for reservoir:

$$N(\xi) + i \cdot T(\xi) = \sigma_{yy}(\xi) + i \cdot \tau_{xy}(\xi) \qquad \text{for} \qquad |\xi| < 1 \qquad \eta = 0 \tag{2.10}$$

where $N(\xi) + i\,T(\xi)$ are the tractions acting on the hanging and foot sides from inside the reservoir in the *auxiliary problem* (Figure 1.2). Substituting Eq. (2.4) into Eqs. (1.10), (1.11) and using the result along with Eqs. (2.5), (2.6), (2.8) and (2.9) in Eq. (2.10) for $\theta = 0$ (there is no fault at this stage and global and local coordinate sets coincide) and $\zeta - \bar{\zeta} = 2\eta i = 0$, we obtain a system of $2N$ linear algebraic equations (N complex equations) with respect to $2N$ unknowns, α_n and β_n (n=1,2,...,N) :

$$\begin{aligned}
&\left[\frac{4\cdot a}{E'}\cdot\sqrt{1-\left(\xi_j\right)^2}\cdot\sum_{n=1}^{N}\frac{k\left(\xi_j\right)\cdot\alpha_n + i\cdot m\left(\xi_j\right)\cdot\beta_n}{n}\cdot U_{n-1}\left(\xi_j\right)\right] \cdots \\
&+\sum_{n=1}^{N} a\cdot\frac{\alpha_n\cdot\lambda_1 - i\cdot\beta_n\cdot\mu_1}{n}\cdot\frac{1-\kappa}{4\cdot\mu}\cdot\mathrm{Re}\left[\frac{d}{d\xi_j}G_n\left(\xi_j\right)\right] \cdots \\
&+(-1)\cdot\left[\begin{aligned}&\alpha_p\cdot p\left(\xi_j\right) + \sigma_{yy_remote} + i\cdot\tau_{xy_remote} - \left(\lambda_1 + 2\cdot\mu_1\right)\cdot\varepsilon_{yy_remote} \cdots \\ &+(-1)\cdot\lambda_1\cdot\varepsilon_{xx_remote} - 2\cdot i\cdot\mu_1\cdot\varepsilon_{xy_remote}\end{aligned}\right]
\end{aligned}
= \begin{aligned}&\mathrm{Re}\left[\sum_{n=1}^{N}\left(\alpha_n - i\cdot\beta_n\right)\cdot\left[I_n\left(\xi_j\right) + i\cdot J_n\left(\xi_j\right)\right]\right] \cdots \\ &+ i\cdot\left[\sum_{n=1}^{N}\beta_n\cdot\left[I_n\left(\xi_j\right) + i\cdot J_n\left(\xi_j\right)\right]\right]\end{aligned}$$

where $j = 1, 2 .. N$ is the equation number and we choose Chebyshev's collocation points: (2.11)

$$\xi_j = \cos\left[\frac{\pi\cdot(2\cdot j-1)}{2\cdot N}\right] \tag{2.12}$$

Separating the real and imaginary parts in Eq. (2.11), we obtain

$$\begin{aligned}\sum_{n=1}^{N}\alpha_n\cdot\left[\frac{4\cdot k\left(\xi_j\right)\cdot a}{n\cdot E'}\cdot\sqrt{1-\left(\xi_j\right)^2}\cdot U_{n-1}\left(\xi_j\right) - I_n\left(\xi_j\right) + \frac{\lambda_1}{4\cdot\mu}\cdot\frac{1-\kappa}{n}\cdot\mathrm{Re}\left[\frac{d}{d\xi_j}G_n\left(\xi_j\right)\right]\right] = {}&\alpha_p\cdot p\left(\xi_j\right) + \sigma_{yy\_remote} \cdots \\ &+(-1)\cdot\left(\lambda_1 + 2\cdot\mu_1\right)\cdot\varepsilon_{yy\_remote} - \lambda_1\cdot\varepsilon_{xx\_remote}\end{aligned} \tag{2.13}$$

and

$$\sum_{n=1}^{N}\beta_n\cdot\left[\frac{4\cdot m\left(\xi_j\right)\cdot a}{n\cdot E'}\cdot\sqrt{1-\left(\xi_j\right)^2}\cdot U_{n-1}\left(\xi_j\right) - I_n\left(\xi_j\right) - \frac{\mu_1}{4\cdot\mu}\cdot\frac{1-\kappa}{n}\cdot\mathrm{Re}\left[\frac{d}{d\xi_j}G_n\left(\xi_j\right)\right]\right] = \tau_{xy\_remote} - 2\cdot\mu_1\cdot\varepsilon_{xy\_remote} \tag{2.14}$$

Introduce normalized reservoir thickness:

$$\delta(x)=\frac{h(x)}{a}=\frac{h_{max}}{a}\cdot f_h(x)=\delta_{max}\cdot f_h(x) \qquad \delta_{max}=\frac{h_{max}}{a} \tag{2.15}$$

Introduce normalized reservoir rigidities:

$$\varepsilon_a=2\cdot\frac{\lambda_1+2\cdot\mu_1}{E'} \qquad \varepsilon_a=\frac{2\cdot(1-\nu_1)\cdot(1-\nu^2)}{(1+\nu_1)\cdot(1-2\cdot\nu_1)}\cdot\frac{E_1}{E} \tag{2.16a}$$

$$\varepsilon_b=4\cdot\frac{\mu_1}{E'} \qquad \varepsilon_b=2\cdot\frac{1-\nu^2}{1+\nu_1}\cdot\frac{E_1}{E} \tag{2.16b}$$

$$\varepsilon_c=\frac{\lambda_1}{4\cdot\mu}\cdot(1-\kappa) \qquad \varepsilon_c=\frac{E_1\cdot\nu_1}{(1+\nu_1)\cdot(1-2\cdot\nu_1)}\cdot\frac{1+\nu}{2\cdot E}\cdot(-2+4\cdot\nu) \tag{2.16c}$$

$$\varepsilon_d=\frac{\mu_1}{4\cdot\mu}\cdot(1-\kappa) \qquad \varepsilon_d=\frac{E_1}{2\cdot(1+\nu_1)}\cdot\frac{1+\nu}{2\cdot E}\cdot(-2+4\cdot\nu) \tag{2.16d}$$

Below we rewrite (2.13) and (2.14) using these notations and quantities

$$\varepsilon_k=\frac{1-\nu^2}{(1+\nu_1)\cdot(1-2\cdot\nu_1)}\cdot\left[\nu_1-(1-\nu_1)\cdot\frac{\nu}{1-\nu}\right]\cdot\frac{E_1}{E} \tag{2.16e}$$

$$\varepsilon_l=\frac{1-\nu^2}{(1+\nu_1)\cdot(1-2\cdot\nu_1)}\cdot\left[(1-\nu_1)-\nu_1\cdot\frac{\nu}{1-\nu}\right]\cdot\frac{E_1}{E} \tag{2.16f}$$

$$\varepsilon_m=\frac{1+\nu}{1+\nu_1}\cdot\frac{E_1}{E} \tag{2.16g}$$

which represent the coefficients on the right hand sides of (2.13) and (2.14) after the remote stresses are replaced by remote strains through the Hooke's law. We will also need quantities

$$\frac{4\cdot k(\xi_j)\cdot a}{E'}=4\cdot\frac{\lambda_1+2\cdot\mu_1}{h\cdot E'}\cdot a=\frac{4\cdot a}{h}\cdot\varepsilon_a=\frac{2\cdot\varepsilon_a(E,E_1,\nu,\nu_1)}{\delta} \tag{2.17}$$

$$\frac{4\cdot m(\xi_j)\cdot a}{E'}=\frac{4\cdot\mu_1}{h\cdot E'}\cdot a=\frac{a}{h}\cdot\left(2\cdot\frac{1-\nu^2}{1+\nu_1}\cdot\frac{E_1}{E}\right)=\frac{\varepsilon_b}{\delta} \tag{2.18}$$

Then, using Eqs. (2.15) through (2.18) in Eqs. (2.13) and (2.14), we obtain,

$$\sum_{n=1}^{N} \alpha_n \cdot \left[\frac{2 \cdot \varepsilon_a}{n \cdot \delta} \cdot \sqrt{1 - (\xi_j)^2} \cdot U_{n-1}(\xi_j) - I_n(\xi_j) + \varepsilon_c \cdot \mathrm{Re}\left[\frac{d}{d\xi_j} G_n(\xi_j) \right] \right] = \alpha_p \cdot p\xi_j + (1 - \varepsilon_l) \cdot \sigma_{yy\_remote} - \varepsilon_k \cdot \sigma_{xx\_remote} \tag{2.19}$$

and

$$\sum_{n=1}^{N} \beta_n \cdot \left[\frac{\varepsilon_b}{n \cdot \delta} \cdot \sqrt{1 - (\xi_j)^2} \cdot U_{n-1}(\xi_j) - I_n(\xi_j) - \varepsilon_d \cdot \mathrm{Re}\left[\frac{d}{d\xi_j} G_n(\xi_j) \right] \right] = (1 - \varepsilon_m) \cdot \tau_{xy\_remote} \tag{2.20}$$

Equation (2.19) can be now represented in the matrix form:

$$\sum_{n=1}^{N} A_{j,n} \cdot \varepsilon_{\alpha_n} = R_{\alpha_j} \qquad \text{or} \qquad A \cdot \varepsilon_\alpha = R_\alpha \tag{2.21}$$

where comma does not mean differentiation but rather separates the matrix indices (standard Mathcad 8 Pro notation). Here

$$A_{j,n} = \frac{2 \cdot \varepsilon_a}{n \cdot \delta} \cdot \sqrt{1 - (\xi_j)^2} U_{n-1}(\xi_j) - I_n(\xi_j) + \varepsilon_c \cdot \mathrm{Re}\left[\frac{d}{d\xi_j} G_n(\xi_j) \right] \tag{2.22}$$

Normalized vector of α coefficients is:

$$\varepsilon_{\alpha_n} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ .. \\ \alpha_N \end{bmatrix} \cdot \frac{1}{\sigma_0} \tag{2.23}$$

The normalized "right hand side" *in (2.21) containing external stress and inclusion* response:

$$R_{\alpha_j} = \frac{\alpha_p \cdot p\xi_j + (1 - \varepsilon_l) \cdot \sigma_{yy\_remote} - \varepsilon_k \cdot \sigma_{xx\_remote}}{\sigma_0} \tag{2.24}$$

Similarly, Eq. (2.20) can be represented in the matrix form as :

$$\sum_{n=1}^{N} B_{j,n} \cdot \varepsilon_{\beta_n} = R_{\beta_j} \qquad \text{or} \qquad B \cdot \varepsilon_{\beta} = R_{\beta} \tag{2.25}$$

where

$$B_{j,n} = \frac{\varepsilon_b}{n \cdot \delta} \cdot \sqrt{1 - (\xi_j)^2} \cdot U_{n-1}(\xi_j) - I_n(\xi_j) - \varepsilon_d \cdot \mathrm{Re}\left[\frac{d}{d\xi_j} G_n(\xi_j)\right] \tag{2.26}$$

$$\varepsilon_{\beta_n} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ .. \\ \beta_N \end{bmatrix} \cdot \frac{1}{\sigma_0} \tag{2.27}$$

$$R_{\beta_j} = \frac{(1 - \varepsilon_m)\tau_{xy\_remote}}{\sigma_0} \tag{2.28}$$

2.3. Defining Auxiliary Formulae for Calculations

Relative reservoir thickness:

$$\delta_{max}(h_{max}, a) := \frac{h_{max}}{a} \tag{2.29}$$

$$\delta(h_{max}, f_h, a, \xi) := \delta_{max}(h_{max}, a) \cdot f_h(\xi) \tag{2.30}$$

Normalized rigidity of reservoir material:

$$\varepsilon_a(E, E_1, \nu, \nu_1) := \frac{2 \cdot (1-\nu_1) \cdot (1-\nu^2)}{(1+\nu_1) \cdot (1-2 \cdot \nu_1)} \cdot \frac{E_1}{E} \tag{2.31}$$

$$\varepsilon_b(E, E_1, \nu, \nu_1) := 2 \cdot \frac{1-\nu^2}{1+\nu_1} \cdot \frac{E_1}{E} \tag{2.32}$$

$$\varepsilon_c(E, E_1, \nu, \nu_1) := \frac{E_1 \cdot \nu_1}{(1+\nu_1) \cdot (1-2 \cdot \nu_1)} \cdot \frac{1+\nu}{E} \cdot (2 \cdot \nu - 1) \tag{2.33}$$

$$\varepsilon_d(E, E_1, \nu, \nu_1) := \frac{E_1}{2 \cdot (1+\nu_1)} \cdot \frac{1+\nu}{E} \cdot (2 \cdot \nu - 1) \tag{2.34}$$

$$\varepsilon_k(E, E_1, \nu, \nu_1) := \frac{1-\nu^2}{(1+\nu_1) \cdot (1-2 \cdot \nu_1)} \cdot \left[\nu_1 - (1-\nu_1) \cdot \frac{\nu}{1-\nu}\right] \cdot \frac{E_1}{E} \tag{2.35}$$

$$\varepsilon_l(E, E_1, \nu, \nu_1) := \frac{1-\nu^2}{(1+\nu_1) \cdot (1-2 \cdot \nu_1)} \cdot \left[(1-\nu_1) - \nu_1 \cdot \frac{\nu}{1-\nu}\right] \cdot \frac{E_1}{E} \tag{2.36}$$

$$\varepsilon_m(E, E_1, \nu, \nu_1) := \frac{1+\nu}{1+\nu_1} \cdot \frac{E_1}{E} \tag{2.37}$$

Counters of collocation points:

$j := 1, 2 .. N$ $\qquad$ $n := 1, 2 .. N$

Auxiliary functions:

$$g(\zeta, n) := \frac{\left(\zeta - \sqrt{\zeta - 1} \cdot \sqrt{\zeta + 1}\right)^n}{\sqrt{\zeta - 1} \cdot \sqrt{\zeta + 1}} \tag{2.38}$$

$$I(\zeta,n) := \mathrm{Re}(g(\zeta,n)) \tag{2.39}$$

$$J(\zeta,n) := \mathrm{Im}(g(\zeta,n)) \tag{2.40}$$

$$f(\zeta,n) := \left(\frac{n}{\sqrt{\zeta-1}\cdot\sqrt{\zeta+1}} + \frac{\zeta}{\zeta^2-1}\right)\cdot g(\zeta,n) \tag{2.41}$$

$$G(\xi,n) := \mathrm{Re}\left[\left(\xi - i\cdot\sqrt{1+\xi}\cdot\sqrt{1-\xi}\right)^{n-1}\cdot\left(1 + i\cdot\frac{\xi}{\sqrt{1+\xi}\cdot\sqrt{1-\xi}}\right)\right] \tag{2.42}$$

Chebyshev's polynomia of the second kind

$$U(\zeta,n) := \frac{\sin((n+1)\cdot \mathrm{acos}(\zeta))}{\sin(\mathrm{acos}(\zeta))} \tag{2.43}$$

Coordinates of Chebyshev's collocation points along the reservoir:

$$\xi_j := \cos\left[\pi\cdot\frac{(2\cdot j-1)}{2\cdot N}\right] \tag{2.44}$$

3. Computing Chebyshev coefficients

3.1. Formation of Vector α

IMPORTANT!
Functions $f_h(x)$ and $f_{\Delta p}(x)$ have to be redefined every time the reservoir shape and distribution of pressure depletion/injection inside the reservoir are changed. After redefining them, all the calculations based on these functions have to be repeated and this is why some parts of the code are repeated as well.

$$A_{1_{j,n}} := \frac{\sqrt{1-(\xi_j)^2}\cdot U(\xi_j, n-1)}{f_h(\xi_j)\cdot n} \tag{3.1}$$

$$I_{1_{j,n}} := I(\xi_j, n) \tag{3.2}$$

$$G_{1_{j,n}} := G(\xi_j, n) \tag{3.3}$$

$$A(E, E_1, \nu, \nu_1, h_{max}, a) := \frac{2\cdot\varepsilon_a(E, E_1, \nu, \nu_1)}{\delta_{max}(h_{max}, a)}\cdot A_1 - I_1 + \varepsilon_c(E, E_1, \nu, \nu_1)\cdot G_1 \tag{3.4}$$

$$R_{\alpha 1_j} := f_{\Delta p}(\xi_j) \tag{3.5}$$

$$R_{\alpha 2_j} := 1 \tag{3.6}$$

$$R_\alpha(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := R_{\alpha 1}\cdot\frac{\alpha_p\cdot\Delta p_{max}}{\sigma_0} \dots + R_{\alpha 2}\cdot\frac{(1-\varepsilon_1(E, E_1, \nu, \nu_1))\cdot\sigma_{yy\_remote}(\gamma, \lambda, \sigma_1) \dots + (-1)\cdot\varepsilon_k(E, E_1, \nu, \nu_1)\cdot\sigma_{xx\_remote}(\gamma, \lambda, \sigma_1) + \alpha_p\cdot p_0}{\sigma_0} \tag{3.7}$$

To compute ε_α from Eq. (2.23), we use [lsolve] operator of Mathcad 8 Pro:

$$\varepsilon_\alpha(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \text{lsolve}(A(E, E_1, \nu, \nu_1, h_{max}, a), R_\alpha(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)) \tag{3.8}$$

$$\alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \varepsilon_\alpha(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)\cdot\sigma_0 \tag{3.9}$$

$$\alpha := \alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) \tag{3.10}$$

$$\alpha_0 := \alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max\_0}, p_0, \alpha_p) \tag{3.10a}$$

NOTE: α_0 is needed to compute initial or residual stress distributions in Section 4.4.

3.2. Formation of Vector β

$$B1_{j,n} := \frac{\sqrt{1-(\xi_j)^2}\cdot U(\xi_j, n-1)}{f_h(\xi_j)\cdot n} \tag{3.11}$$

$$B(E, E_1, \nu, \nu_1, h_{max}, a) := \frac{\varepsilon_b(E, E_1, \nu, \nu_1)}{\delta_{max}(h_{max}, a)}\cdot B1 - I_1 - \varepsilon_d(E, E_1, \nu, \nu_1)\cdot G_1 \tag{3.12}$$

$$R_{\beta1_j} := 1 \tag{3.13}$$

$$R_\beta(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := R_{\beta1}\cdot\frac{(1-\varepsilon_m(E, E_1, \nu, \nu_1))\cdot\tau_{xy\_remote}(\gamma, \lambda, \sigma_1)}{\sigma_0} \tag{3.14}$$

$$\varepsilon_\beta(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \text{lsolve}(B(E, E_1, \nu, \nu_1, h_{max}, a), R_\beta(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)) \tag{3.15}$$

$$\beta'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \varepsilon_\beta(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)\cdot\sigma_0 \tag{3.16}$$

$$\beta := \beta'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) \tag{3.17}$$

$$\beta_0 := \beta'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max\_0}, p_0, \alpha_p) \tag{3.17a}$$

NOTE: β_0 is needed to compute initial or residual stress distributions in Section 4.4.

4. Stresses Outside and Inside the Reservoir

4.1. Stresses Outside the Reservoir in Auxiliary Problem (Figure 1.2)

In the reservoir coordinate set:

$$\sigma_{yy}(\zeta,\alpha,\beta) := \mathrm{Re}\left[\sum_{n=1}^{N}\left(\alpha_n - i\cdot\beta_n\right)\cdot g(\zeta,n) + i\cdot\sum_{n=1}^{N}\beta_n\cdot g(\zeta,n)\ \ldots \atop +\frac{1}{2}\cdot\left(\zeta-\overline{\zeta}\right)\cdot\sum_{n=1}^{N}\left(\alpha_n - i\cdot\beta_n\right)\cdot f(\zeta,n)\right] \tag{4.1}$$

$$\sigma_{xx}(\zeta,\alpha,\beta) := 2\cdot\mathrm{Re}\left[\sum_{n=1}^{N}\left(\alpha_n - i\cdot\beta_n\right)\cdot g(\zeta,n)\right] - \sigma_{yy}(\zeta,\alpha,\beta) \tag{4.2}$$

$$\tau_{xy}(\zeta,\alpha,\beta) := \mathrm{Im}\left[i\cdot\sum_{n=1}^{N}\beta_n\cdot g(\zeta,n) + \frac{1}{2}\cdot\left(\zeta-\overline{\zeta}\right)\cdot\sum_{n=1}^{N}\left(\alpha_n - i\cdot\beta_n\right)\cdot f(\zeta,n)\right] \tag{4.3}$$

In the inclined (fault) coordinate set:

$$\sigma_{x'x'\_0}(\zeta,\theta,\alpha,\beta) := \sigma_{xx}(\zeta,\alpha,\beta)\cdot\cos(\theta)^2 + \sigma_{yy}(\zeta,\alpha,\beta)\cdot\sin(\theta)^2\ \ldots \atop +2\cdot\tau_{xy}(\zeta,\alpha,\beta)\cdot\sin(\theta)\cdot\cos(\theta) \tag{4.4}$$

$$\sigma_{y'y'\_0}(\zeta,\theta,\alpha,\beta) := \sigma_{yy}(\zeta,\alpha,\beta)\cdot\cos(\theta)^2\ \ldots \atop +\sigma_{xx}(\zeta,\alpha,\beta)\cdot\sin(\theta)^2 - 2\cdot\tau_{xy}(\zeta,\alpha,\beta)\cdot\sin(\theta)\cdot\cos(\theta) \tag{4.5}$$

$$\tau_{x'y'\_0}(\zeta,\theta,\alpha,\beta) := \frac{\sigma_{yy}(\zeta,\alpha,\beta) - \sigma_{xx}(\zeta,\alpha,\beta)}{2}\cdot\sin(2\cdot\theta) + \tau_{xy}(\zeta,\alpha,\beta)\cdot\cos(2\cdot\theta) \tag{4.6}$$

4.2. Remote In-Situ Stresses (Figure 1.2)

In the inclined (fault) coordinate set:

$$\sigma_{x'x'\_remote}\left(\theta,\gamma,\lambda,\sigma_1\right) := \sigma_1\cdot\sin(\theta+\gamma)^2 + \sigma_3\left(\lambda,\sigma_1\right)\cdot\cos(\theta+\gamma)^2 \tag{4.7}$$

$$\sigma_{y'y'\_remote}\left(\theta,\gamma,\lambda,\sigma_1\right) := \sigma_1\cdot\cos(\theta+\gamma)^2 + \sigma_3\left(\lambda,\sigma_1\right)\cdot\sin(\theta+\gamma)^2 \tag{4.8}$$

$$\tau_{x'y'\_remote}\left(\theta,\gamma,\lambda,\sigma_1\right) := \left(\sigma_1 - \sigma_3\left(\lambda,\sigma_1\right)\right)\cdot\sin(\theta+\gamma)\cdot\cos(\theta+\gamma) \tag{4.9}$$

4.3. Total Stresses Outside the Reservoir in the case of Zero Residual Stresses

In the inclined (fault) coordinate set:

$$\sigma_{x'x'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta) := \sigma_{x'x'\_0}(\zeta,\theta,\alpha,\beta) + \sigma_{x'x'\_remote}(\theta,\gamma,\lambda,\sigma_1) \quad (4.10)$$

$$\sigma_{y'y'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta) := \sigma_{y'y'\_0}(\zeta,\theta,\alpha,\beta) + \sigma_{y'y'\_remote}(\theta,\gamma,\lambda,\sigma_1) \quad (4.11)$$

$$\tau_{x'y'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta) := \tau_{x'y'\_0}(\zeta,\theta,\alpha,\beta) + \tau_{x'y'\_remote}(\theta,\gamma,\lambda,\sigma_1) \quad (4.12)$$

4.4. Residual Stresses

NOTE: These total stresses are only needed to compute the residual stresses in the next subsection 4.5. After that they will be redefined for the general case of residual stresses (i.e., not necessarily zero).

$$\sigma_{x'x'\_residual}(\zeta,\theta,\gamma,\lambda,\sigma_1) := 0\cdot Pa \quad (4.12a)$$

$$\sigma_{y'y'\_residual}(\zeta,\theta,\gamma,\lambda,\sigma_1) := 0\cdot Pa \quad (4.12b)$$

$$\tau_{x'y'\_residual}(\zeta,\theta,\gamma,\lambda,\sigma_1) := 0\cdot Pa \quad (4.12c)$$

$$\sigma_{x'x'\_residual}(\zeta,\theta,\gamma,\lambda,\sigma_1) := \sigma_{x'x'\_remote}(\theta,\gamma,\lambda,\sigma_1) - \sigma_{x'x'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha_0,\beta_0) \quad (4.12d)$$

$$\sigma_{y'y'\_residual}(\zeta,\theta,\gamma,\lambda,\sigma_1) := \sigma_{y'y'\_remote}(\theta,\gamma,\lambda,\sigma_1) - \sigma_{y'y'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha_0,\beta_0) \quad (4.12e)$$

$$\tau_{x'y'\_residual}(\zeta,\theta,\gamma,\lambda,\sigma_1) := \tau_{x'y'\_remote}(\theta,\gamma,\lambda,\sigma_1) - \tau_{x'y'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha_0,\beta_0) \quad (4.12f)$$

NOTE: In Fault 99, the user currently has two options:

(1) residual stresses are zero;
(2) initial stress state is uniform (lithostatic).

Accordingly the user have to enable/disable one or another pairs of these expressions. If the user wishes to consider different residual/initial stress distribution, the corresponding expressions have to be typed in on the place of those provided here.

4.5. Total Stresses Outside the Reservoir (General Case of Residual Stresses)

$$\sigma_{x'x'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta) := \sigma_{x'x'\_0}(\zeta,\theta,\alpha,\beta) + \sigma_{x'x'\_remote}(\theta,\gamma,\lambda,\sigma_1) + \sigma_{x'x'\_residual}(\zeta,\theta,\gamma,\lambda,\sigma_1) \quad (4.12g)$$

$$\sigma_{y'y'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta) := \sigma_{y'y'\_0}(\zeta,\theta,\alpha,\beta) + \sigma_{y'y'\_remote}(\theta,\gamma,\lambda,\sigma_1) + \sigma_{y'y'\_residual}(\zeta,\theta,\gamma,\lambda,\sigma_1) \quad (4.12h)$$

$$\tau_{x'y'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta) := \tau_{x'y'\_0}(\zeta,\theta,\alpha,\beta) + \tau_{x'y'\_remote}(\theta,\gamma,\lambda,\sigma_1) + \tau_{x'y'\_residual}(\zeta,\theta,\gamma,\lambda,\sigma_1) \quad (4.12i)$$

4.6. Stresses Inside the Reservoir

From Eq. (2.1), displacement discontinuities across the reservoir are:

$$\Delta v(\xi) := \mathrm{Re}\left[\frac{4\cdot a\cdot\left(1-\nu^2\right)}{E}\cdot\sqrt{1-\xi^2}\cdot\sum_{n=1}^{N}\frac{1}{n}\cdot\left(\alpha_n + i\cdot\beta_n\right)\cdot U(\xi,n-1)\right] \quad (4.13)$$

$$\Delta u(\xi) := \mathrm{Im}\left[\frac{4\cdot a\cdot\left(1-\nu^2\right)}{E}\cdot\sqrt{1-\xi^2}\cdot\sum_{n=1}^{N}\frac{1}{n}\cdot\left(\alpha_n + i\cdot\beta_n\right)\cdot U(\xi,n-1)\right] \quad (4.14)$$

From Eqs. (1.1) and (1.2),

$$\sigma_{yy\_res}(\xi) := \frac{E_1\cdot\left(1-\nu_1\right)}{\left(1+\nu_1\right)\cdot\left(1-2\cdot\nu_1\right)}\cdot\frac{\Delta v(\xi)}{h_{max}\cdot f_h(\xi)} + \varepsilon_c\left(E,E_1,\nu,\nu_1\right)\cdot\sum_{n=1}^{N}\alpha_n\cdot G(\xi,n)\ \ldots$$
$$+\varepsilon_k\left(E,E_1,\nu,\nu_1\right)\cdot\sigma_{xx\_remote}\left(\gamma,\lambda,\sigma_1\right) + \varepsilon_l\left(E,E_1,\nu,\nu_1\right)\cdot\sigma_{yy\_remote}\left(\gamma,\lambda,\sigma_1\right) - \alpha_p\cdot\left(f_{\Delta p}(\xi)\cdot\Delta p_{max} + p_0\right) \quad (4.15)$$

$$\sigma_{xx\_res}(\xi) := \frac{E_1\cdot\nu_1}{\left(1+\nu_1\right)\cdot\left(1-2\cdot\nu_1\right)}\cdot\frac{\Delta v(\xi)}{h_{max}\cdot f_h(\xi)} + \frac{E_1\cdot\left(1-\nu_1\right)}{\left(1+\nu_1\right)\cdot\left(1-2\cdot\nu_1\right)}\cdot\frac{1+\nu}{E}\cdot(2\cdot\nu-1)\cdot\sum_{n=1}^{N}\alpha_n\cdot G(\xi,n)\ \ldots$$
$$+\varepsilon_l\left(E,E_1,\nu,\nu_1\right)\cdot\sigma_{xx\_remote}\left(\gamma,\lambda,\sigma_1\right) + \varepsilon_k\left(E,E_1,\nu,\nu_1\right)\cdot\sigma_{yy\_remote}\left(\gamma,\lambda,\sigma_1\right) - \alpha_p\cdot\left(f_{\Delta p}(\xi)\cdot\Delta p_{max} + p_0\right) \quad (4.16)$$

$$\tau_{xy\_res}(\xi) := \frac{E_1}{2\cdot\left(1+\nu_1\right)}\cdot\frac{\Delta u(\xi)}{h_{max}\cdot f_h(\xi)} - \varepsilon_d\left(E,E_1,\nu,\nu_1\right)\cdot\sum_{n=1}^{N}\beta_n\cdot G(\xi,n) + \varepsilon_m\left(E,E_1,\nu,\nu_1\right)\cdot\tau_{xy\_remote}\left(\gamma,\lambda,\sigma_1\right) \quad (4.17)$$

5. Criterion of Fault Slip

Mohr-Coulomb strength criterion is employed.

Shear strength along the fault is:

$$\tau_f(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi,C,p_f) := \begin{vmatrix} \left|\sigma_{y'y'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta) + p_f\right|\cdot\tan(\phi) + C \text{ if } \left(\sigma_{y'y'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta) + p_f\right) \le 0\cdot\text{Pa} \\ C \text{ otherwise} \end{vmatrix} \tag{5.1}$$

Coulomb shear stress along the fault is defined as the difference between the available shear stress and friction resistance (accounting for cohesion would constitute the Mohr-Coulomb stresses which is called driving shear stress in the next section):

$$\tau_{Coulomb}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi,p_f) := \begin{vmatrix} \left|\tau_{x'y'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta)\right| \text{ if } \left(\sigma_{y'y'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta) + p_f\right) \ge 0\cdot\text{Pa} \\ \left|\tau_{x'y'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta)\right| - \left|\sigma_{y'y'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta) + p_f\right|\cdot\tan(\phi) \text{ otherwise} \end{vmatrix} \tag{5.2}$$

Friction required to maintain stability for a given cohesion is:

$$\phi\_req(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,C,p_f) := \begin{vmatrix} \text{atan}\left[\dfrac{\left|\tau_{x'y'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta)\right| - C}{\left|\sigma_{y'y'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta) + p_f\right|}\right] \text{ if } \left(\sigma_{y'y'}(\zeta,\theta,\gamma,\lambda,\sigma_1,\alpha,\beta) + p_f\right) \le 0\cdot\text{Pa} \\ \dfrac{\pi}{2} \text{ otherwise} \end{vmatrix} \tag{5.3}$$

6. Fault Positions and Data Points along the Fault

Three fault positions are considered (Figure 6.1). Given the fault inclination θ, they are defined by the points, ξ_f, where the fault intersects the reservoir axis:

$\xi_f := \begin{bmatrix} -2 \\ 0 \end{bmatrix}$ Non end faults **NOTE: The user should define the desired fault positions here. The user can define more than two faults by adding the number of rows in this array.**

$\xi_{f\_end} := -1$ End fault

$\gamma = 0 \cdot deg$ Fault inclination, which is defined in Section 1.

$j := 1 .. last(\xi_f)$ Non end fault number

$M := 199$ $M1 := M + 1$ Number of data points used for plotting along the fault lines

$i := 1, 2 .. M + 1$ $i1 := 1, 2 .. M1 + 1$ Data point counter

$\xi'_{min} := -4$ $\xi'_{max} := 4$ Minimum and maximum coordinates along the fault lines

$\Delta\xi' := \dfrac{h_{max}}{2 \cdot a}$

NOTE: $\Delta\xi' = \dfrac{\delta_{max}}{2}$ **[see (2.15)] but can be redefined by the user along with other ξ'–parameters used here for identifying the plotting region.**

$\xi'_{min\_1} := -4$ $\xi'_{max\_2} := 4$ Minimum and maximum coordinates along the end fault line

$\xi'_{max\_1} := -\Delta\xi'$ $\xi'_{min\_2} := \Delta\xi'$ Minimum and maximum coordinates of the segment along the end fault line excluded from computation.

NOTE: For the end fault, the points, $|\xi'| < \Delta\xi' = h/(2a)$, are avoided because of the inherent singularity near the reservoir end. The solution in this region is linearly interpolated between $\xi' = \Delta\xi'$, $\eta' = 0$ and $\xi' = -\Delta\xi'$, $\eta' = 0$.

Coordinates of data points along the fault lines are defined by:

$$\xi'_{i1} := \xi'_{min} + \frac{\xi'_{max} - \xi'_{min}}{M1} \cdot (i1 - 1)$$

$$\xi'_{end_i} := \left| \begin{array}{ll} \left[\xi'_{min\_1} + \dfrac{\xi'_{max\_1} - \xi'_{min\_1}}{\frac{M-1}{2}} \cdot (i - 1) \right] & \text{if } i \le \dfrac{M+1}{2} \\ \left[\xi'_{min\_2} + \dfrac{\xi'_{max\_2} - \xi'_{min\_2}}{\frac{M-1}{2}} \cdot \left(i - \dfrac{M+3}{2} \right) \right] & \text{otherwise} \end{array} \right.$$

Coordinates of data points along the reservoir are defined by:

$res_i := -1 + \frac{2}{M} \cdot (i - 1)$

$x_{i1,j} := \xi_{f_j} + \xi'_{i1} \cdot \cos(\theta)$

$x_{end_i} := \xi_{f\_end} + \xi'_{end_i} \cdot \cos(\theta)$

$y_{i1,j} := \xi'_{i1} \cdot \sin(\theta)$

$y_{end_i} := \xi'_{end_i} \cdot \sin(\theta)$

$\zeta_{i1,j} := x_{i1,j} + i \cdot y_{i1,j}$ Complex coordinates of data points along the fault lines in the reservoir coordinate set

$\zeta_{end_i} := x_{end_i} + i \cdot y_{end_i}$ Complex coordinates of data points along the end fault line in the reservoir coordinate set

$x_{min} := \min(x) \quad y_{min} := \min(y)$ Minimum values of fault lines coordinates (to set limits for plots)

$x_{max} := \max(x) \quad y_{max} := \max(y)$ *Maximum values of fault lines coordinates (to set limits for plots)*

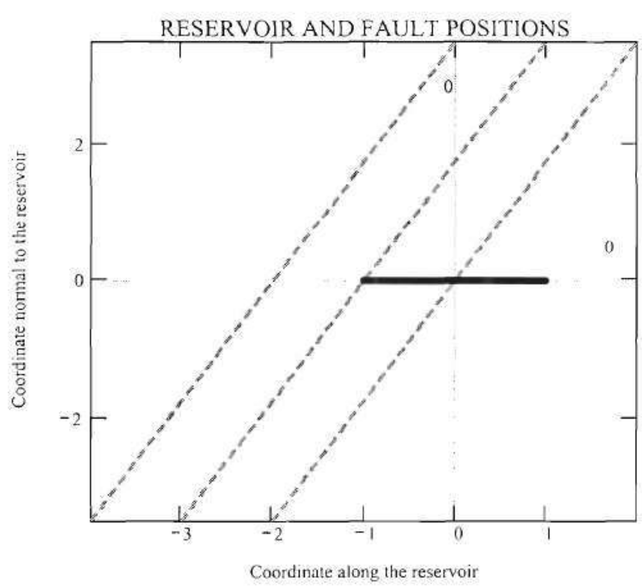


Figure 6.1.

Note: If the user wishes to consider only one fault position, this option is available by choosing either one of ξ_f values defined above (see example in Figure 6.2).

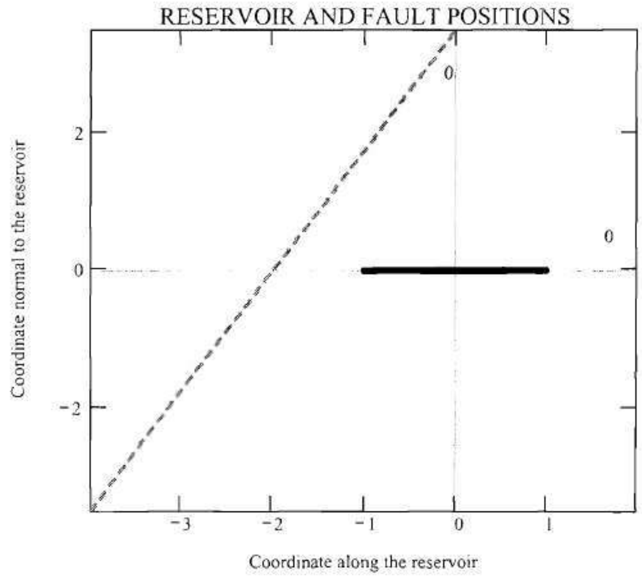


Figure 6.2.

7. Parametric Analysis of Driving Shear Stress (Fault on the Verge of Instability)

For the chosen reference parameters, Coulomb shear stress, $\tau_{Coulomb}$, is calculated for different depletion/injection pressures along the fault line. Coulomb shear stress, $\tau_{Coulomb}$, given by Eq. (5.2) is defined as absolute value of shear stress minus tan(ϕ) times the absolute value of normal stress along the fault line. Coulomb shear stress minus the cohesion gives the *driving* shear stress, i.e., $\tau_{driving} = |\tau| - |\sigma_N| \tan(\phi) - C$. If the driving shear stress is positive, i.e., $\tau_{driving} > 0$, it implies that the fault may reactivate at this point.

Two extreme cases and one general case are considered below. First, in Section 7.1, the friction angle, ϕ, of the fault gouge material is chosen as zero and the cohesion required to maintain the stability of the fault line is calculated at each point along the fault line by equating the driving shear stress to zero:

$$\tau_{driving} = 0 \text{ or } |\tau| - C = 0 \tag{7.1}$$

The maximum value of the cohesion thus obtained along the fault line is chosen as the cohesion for that fault line.

Second, in Section 7.2, the cohesion of the fault gouge material is chosen as zero and the friction angle required to maintain the stability of the fault line is calculated at each point along the fault line by equating the driving shear stress to zero:

$$\tau_{driving} = 0 \text{ or } |\tau| - |\sigma| \cdot \tan(\phi) = 0 \tag{7.2}$$

The maximum value of the friction angle thus obtained along the fault line is chosen as the friction angle for that fault line.

Finally, in Section 7.3, arbitrary friction and cohesion can be assigned by the user.

To see the effect of any individual parameter, one can change the parameter specified in the Section 1 and observe the driving shear stress distribution along the fault line in this section.

NOTE: The shape of the reservoir, $f_h(x)$, and the distribution of pressure depletion/injection, $f_{\Delta p}(\xi)$, are defined in Section 1. Thereby, if the user wants to see the effect of changing the shape of the reservoir or the depletion/injection strategy, then these functions have to be redefined in Section 1.

The choice of depletion/injection strategy, $f_{\Delta p}(\xi)$, may appreciably affect the driving shear stress distribution and is tested in Section 8.

7.1. Zero Friction

Choose friction angle as zero and calculate the driving shear stress, $\tau_{driving} = \tau - \sigma_N \tan(\phi) - C$, and strength, $\tau_f = \sigma_N \tan(\phi) + C$, along different fault lines for the reference parameters (Section 1):

$$\Delta p_{max} := 0 \cdot p_0 \qquad \phi_{no\_friction_j} := 0 \cdot deg \qquad \phi_{no\_friction\_end} := 0 \cdot deg$$

First compute the coefficients α's and β's (Section 3.1 and 3.2):

$$\alpha := \alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) \qquad \beta := \beta'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$$

Coulomb shear stress along the fault lines before depletion/injection is defined by:

$$\tau_{Clmb\_no\_friction\_1_{i1,j}} := \tau_{Coulomb}(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction_j}, p_f)$$

$$\tau_{Clmb\_no\_friction\_end\_1_i} := \tau_{Coulomb}(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction\_end}, p_f)$$

Cohesion required to maintain stability of the fault before depletion/injection assuming friction to be zero is calculated by equating driving shear stress to zero, i.e., by solving $|\tau| - C = 0$:

$$C_{no\_friction_j} := \begin{cases} \max\left(\tau_{Clmb\_no\_friction\_1}^{\langle j \rangle}\right) & \text{if } \max\left(\tau_{Clmb\_no\_friction\_1}^{\langle j \rangle}\right) > 0 \cdot Pa \\ 0 \cdot Pa & \text{otherwise} \end{cases}$$

$$C_{no\_friction\_end} := \begin{cases} \max(\tau_{Clmb\_no\_friction\_end\_1}) & \text{if } \max(\tau_{Clmb\_no\_friction\_end\_1}) > 0 \cdot Pa \\ 0 \cdot Pa & \text{otherwise} \end{cases}$$

$$C_{no\_friction} = \begin{bmatrix} 11.57 \\ 11.98 \end{bmatrix} \cdot MPa \qquad C_{no\_friction\_end} = 18.89 \cdot MPa$$

$$\frac{C_{no\_friction}}{\sigma_0} = \begin{bmatrix} 0.1157 \\ 0.1198 \end{bmatrix} \qquad \frac{C_{no\_friction\_end}}{\sigma_0} = 0.1889$$

Shear strength along the fault lines before depletion/injection is:

$$\tau_{no\_friction\_1_{i1,j}} := \tau_f(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction_j}, C_{no\_friction_j}, p_f)$$

$$\tau_{no\_friction\_end\_1_i} := \tau_f(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction\_end}, C_{no\_friction\_end}, p_f)$$

Now, change reservoir pressure, i.e., *partially* deplete the reservoir: $\Delta p_{max} := -0.5 \cdot p_0$

$$\alpha := \alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) \qquad \beta := \beta'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$$

Coulomb shear stress and strength along the fault lines as a result of the *partial* depletion are:

$\tau_{Clmb\_no\_friction\_2_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction_j}, p_f\right)$

$\tau_{Clmb\_no\_friction\_end\_2_i} := \tau_{Coulomb}\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction\_end}, p_f\right)$

$\tau_{no\_friction\_2_{i1,j}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction_j}, C_{no\_friction_j}, p_f\right)$

$\tau_{no\_friction\_end\_2_i} := \tau_f\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction\_end}, C_{no\_friction\_end}, p_f\right)$

NOTE: The above arrays are used for plotting in Section 8.

Coulomb shear stress and strength along the fault lines as a result of *full* depletion are: $\Delta p_{max} := -1 \cdot p_0$

$\alpha := \alpha'\left(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right)$ $\beta := \beta'\left(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right)$

$\tau_{Clmb\_no\_friction\_3_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction_j}, p_f\right)$

$\tau_{Clmb\_no\_friction\_end\_3_i} := \tau_{Coulomb}\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction\_end}, p_f\right)$

$\tau_{no\_friction\_3_{i1,j}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction_j}, C_{no\_friction_j}, p_f\right)$

$\tau_{no\_friction\_end\_3_i} := \tau_f\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction\_end}, C_{no\_friction\_end}, p_f\right)$

7.2. Zero Cohesion

Choose cohesion as zero and calculate the driving shear stress, $\tau_{driving} = \tau - \sigma_N \tan(\phi) - C$, and strength, $\tau_f = \sigma_N \tan(\phi) + C$, along different fault lines for the reference parameters (Section 1):

$\Delta p_{max} := 0 \cdot p_0$ $C_{no\_cohesion_j} := 0 \cdot Pa$ $C_{no\_cohesion\_end} := 0 \cdot Pa$

First compute the coefficients α's and β's (Section 3.1 and 3.2):

$\alpha := \alpha'\left(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right)$ $\beta := \beta'\left(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right)$

Friction required to maintain stability of the fault before depletion/injection assuming cohesion to be zero is calculated by equating driving shear to zero, i.e., by solving $|\tau| - |\sigma| \cdot \tan(\phi) = 0$:

$\phi_{i1,j} := \phi\_req\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, C_{no\_cohesion_j}, p_f\right)$ $\phi_{no\_cohesion_j} := \max\left(\phi^{\langle j \rangle}\right)$ $\phi_{no\_cohesion} = \begin{bmatrix} 12.56 \\ 12.93 \end{bmatrix} \circ deg$

$\phi_{end_i} := \phi\_req\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, C_{no\_cohesion\_end}, p_f\right)$ $\phi_{no\_cohesion\_end} := \max\left(\phi_{end}\right)$ $\phi_{no\_cohesion\_end} = 17.75 \circ deg$

Coulomb shear stress and strength along the fault lines before depletion/injection are:

$\tau_{Clmb\_no\_cohesion\_1_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion_j}, p_f\right)$

$\tau_{Clmb\_no\_cohesion\_end\_1_i} := \tau_{Coulomb}\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion\_end}, p_f\right)$

$\tau_{no\_cohesion\_1_{i1,j}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion_j}, C_{no\_cohesion_j}, p_f\right)$

$\tau_{no\_cohesion\_end\_1_i} := \tau_f\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion\_end}, C_{no\_cohesion\_end}, p_f\right)$

Now, change reservoir pressure, i.e., *partially* deplete the reservoir: $\Delta p_{max} := -0.5 \cdot p_0$

$\alpha := \alpha'\left(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right)$ $\beta := \beta'\left(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right)$

Coulomb shear stress and strength along the fault lines as a result of the *partial* depletion are:

$\tau_{Clmb\_no\_cohesion\_2_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion_j}, p_f\right)$

$\tau_{Clmb\_no\_cohesion\_end\_2_i} := \tau_{Coulomb}\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion\_end}, p_f\right)$

$\tau_{no\_cohesion\_2_{i1,j}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion_j}, C_{no\_cohesion_j}, p_f\right)$

$\tau_{no\_cohesion\_end\_2_i} := \tau_f\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion\_end}, C_{no\_cohesion\_end}, p_f\right)$

NOTE: The above arrays are used for plotting in Section 8.

Coulomb shear stress and strength along the fault lines as a result of *full* depletion are: $\Delta p_{max} := -1 \cdot p_0$

$\alpha := \alpha'\left(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right)$ $\beta := \beta'\left(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right)$

$\tau_{Clmb\_no\_cohesion\_3_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion_j}, p_f\right)$

$\tau_{Clmb\_no\_cohesion\_end\_3_i} := \tau_{Coulomb}\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion\_end}, p_f\right)$

$\tau_{no\_cohesion\_3_{i1,j}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion_j}, C_{no\_cohesion_j}, p_f\right)$

$\tau_{no\_cohesion\_end\_3_i} := \tau_f\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion\_end}, C_{no\_cohesion\_end}, p_f\right)$

7.3. Arbitrary (User Defined) Friction and Cohesion

Choose friction part of that required without cohesion (7.2) and cohesion part of that required without friction (7.1):

$\Delta p_{max} := 0 \cdot p_0$ $\quad \phi_{arbitrary} := \begin{bmatrix} 6.3 \\ 6.5 \end{bmatrix} \cdot deg$ $\quad \phi_{arbitrary} = \begin{bmatrix} 6.3 \\ 6.5 \end{bmatrix} \circ deg$

$\phi_{arbitrary\_end} := 9 \cdot deg$ $\quad \phi_{arbitrary\_end} = 9 \circ deg$

$C_{arbitrary} := \begin{bmatrix} 5.8 \\ 5.9 \end{bmatrix} \cdot MPa$ $\quad C_{arbitrary} = \begin{bmatrix} 5.8 \\ 5.9 \end{bmatrix} \circ MPa$

$C_{arbitrary\_end} := 9.6 \cdot MPa$ $\quad C_{arbitrary\_end} = 9.6 \circ MPa$

NOTE: If the user wishes to change the above values for friction angle and cohesion, the new values should be typed in directly instead of those highlighted above.

First compute the coefficients α's and β's (Section 3.1 and 3.2):

$\alpha := \alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$ $\quad \beta := \beta'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$

Coulomb shear stress and strength along the fault lines before depletion/injection are:

$\tau_{Clmb\_arbitrary\_1_{i1,j}} := \tau_{Coulomb}(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_j}, p_f)$

$\tau_{Clmb\_arbitrary\_end\_1_i} := \tau_{Coulomb}(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary\_end}, p_f)$

$\tau_{arbitrary\_1_{i1,j}} := \tau_f(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_j}, C_{arbitrary_j}, p_f)$

$\tau_{arbitrary\_end\_1_i} := \tau_f(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary\_end}, C_{arbitrary\_end}, p_f)$

Now, change reservoir pressure, i.e., *partially* deplete the reservoir.

Coulomb shear stress and strength along the fault lines as a result of the *partial* depletion are: $\Delta p_{max} := -0.5 \cdot p_0$

$\alpha := \alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$ $\quad \beta := \beta'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$

$\tau_{Clmb\_arbitrary\_2_{i1,j}} := \tau_{Coulomb}(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_j}, p_f)$

$\tau_{Clmb\_arbitrary\_end\_2_i} := \tau_{Coulomb}\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary\_end}, p_f\right)$

$\tau_{arbitrary\_2_{i1,j}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_j}, C_{arbitrary_j}, p_f\right)$

$\tau_{arbitrary\_end\_2_i} := \tau_f\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary\_end}, C_{arbitrary\_end}, p_f\right)$

NOTE: The above arrays are used for plotting in Section 8.

Coulomb shear stress and strength along the fault lines as a result of *full* depletion are: $\Delta p_{max} := -1 \cdot p_0$

$\alpha := \alpha'\left(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right)$ $\beta := \beta'\left(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right)$

$\tau_{Clmb\_arbitrary\_3_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_j}, p_f\right)$

$\tau_{Clmb\_arbitrary\_end\_3_i} := \tau_{Coulomb}\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary\_end}, p_f\right)$

$\tau_{arbitrary\_3_{i1,j}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_j}, C_{arbitrary_j}, p_f\right)$

$\tau_{arbitrary\_end\_3_i} := \tau_f\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary\_end}, C_{arbitrary\_end}, p_f\right)$

7.4. Plotting

Reference Parameters defined in Section 1:

$E = 1 \cdot 10^{10} \cdot \text{Pa}$ $\qquad \dfrac{2 \cdot a}{h_{max}} = 10$

$\nu = 0.25$

$\nu_1 = 0.35$ $\qquad \gamma = 0 \cdot \text{deg}$

$\dfrac{E_1}{E} = 0.3333$ $\qquad \theta = 60 \cdot \text{deg}$

$\alpha_p = 1$ $\qquad \dfrac{p_f}{\sigma_0} = 0.3$

$\dfrac{p_0}{\sigma_0} = 1$

$\lambda = 0.75$

$\dfrac{\sigma_1}{\sigma_0} = -1$

$\sigma_0 = 100 \cdot \text{MPa}$

IMPORTANT!
Here, the user *must* make sure that $\sigma_0 \neq 0$ because it is further used for normalizing.

$$C := \begin{bmatrix} C_{no\_friction_1} & C_{no\_friction\_end} & C_{no\_friction_2} \\ C_{no\_cohesion_1} & C_{no\_cohesion\_end} & C_{no\_cohesion_2} \\ C_{arbitrary_1} & C_{arbitrary\_end} & C_{arbitrary_2} \end{bmatrix} \qquad C = \begin{bmatrix} 11.57 & 18.89 & 11.98 \\ 0 & 0 & 0 \\ 5.8 & 9.6 & 5.9 \end{bmatrix} \cdot \text{MPa}$$

$$\phi := \begin{bmatrix} \phi_{no\_friction_1} & \phi_{no\_friction\_end} & \phi_{no\_friction_2} \\ \phi_{no\_cohesion_1} & \phi_{no\_cohesion\_end} & \phi_{no\_cohesion_2} \\ \phi_{arbitrary_1} & \phi_{arbitrary\_end} & \phi_{arbitrary_2} \end{bmatrix} \qquad \phi = \begin{bmatrix} 0 & 0 & 0 \\ 12.56 & 17.75 & 12.93 \\ 6.3 & 9 & 6.5 \end{bmatrix} \cdot \text{deg}$$

NOTE: In the above matrices, the columns represent three different fault lines (Section 6) while the rows are related to three different cases, i.e., no friction (i.e., only cohesion), no cohesion (i.e., only friction), arbitrary cohesion and friction. Subscripts "1", "end", and "2" mark remote, end, and crossing faults respectively (Figure 6.1).

$f_{\Delta p\_initial} := 1$ $f_{\Delta p\_final} := 0$ Initial and final pressure distributions in the reservoir (needed only for plotting)

Reference Parameters from Section 1:

$E = 1\cdot 10^{10}\cdot Pa$

$\nu = 0.25$

$\nu_1 = 0.35$

$\frac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\frac{p_0}{\sigma_0} = 1$

$\frac{\sigma_1}{\sigma_0} = -1$

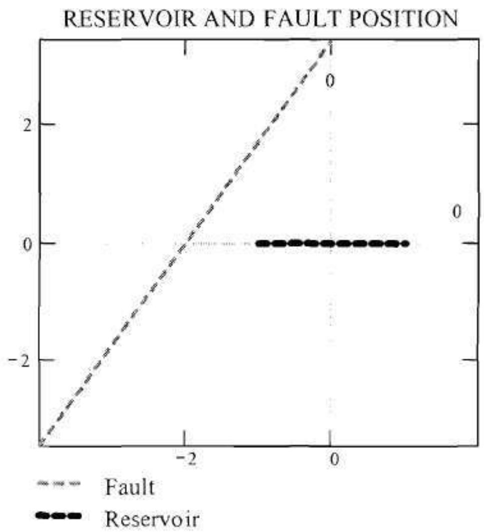
$\lambda = 0.75$

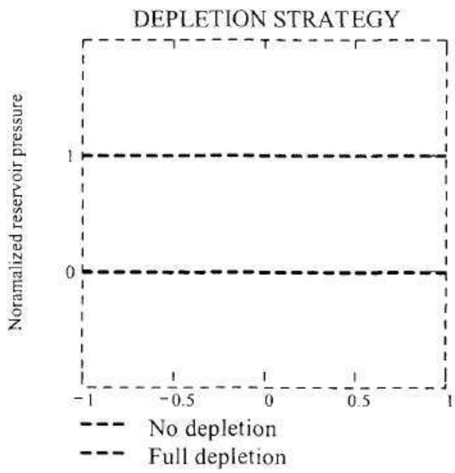
$\frac{p_f}{\sigma_0} = 0.3$

$\frac{2\cdot a}{h_{max}} = 10$

$\gamma = 0\cdot deg$

$\theta = 60\cdot deg$

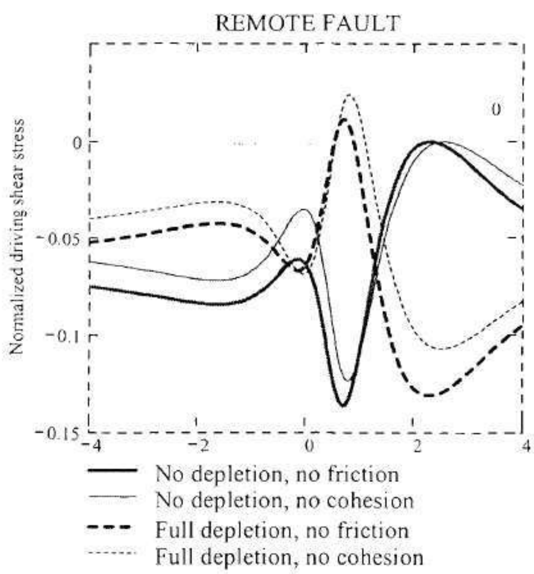




Cohesion $C_{1,1} = 11.57\cdot MPa$, **no friction**

No cohesion, friction $\phi_{2,1} = 12.56\cdot deg$

User defined cohesion $C_{3,1} = 5.8\cdot MPa$ **and friction** $\phi_{3,1} = 6.3\cdot deg$



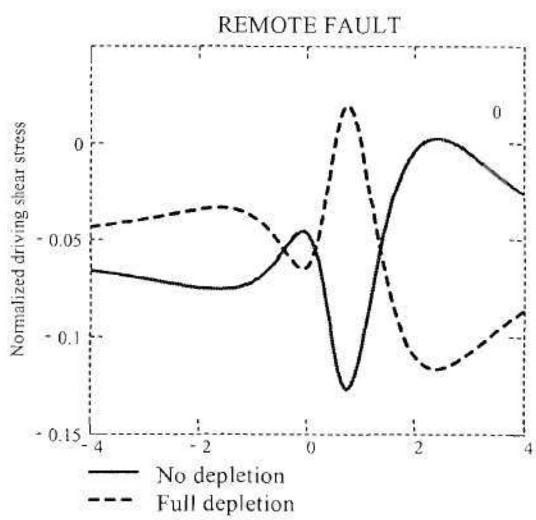


Figure 7.1.

NOTE: The figure on the left is plotted using the friction and cohesion computed in Sections 7.1 and 7.2 for the fault at the verge of instability. The figure on the right is plotted for the parameters identified by the user in Section 7.3.

Reference Parameters from Section 1:

$E = 1 \cdot 10^{10} \cdot Pa$

$\nu = 0.25$

$\nu_1 = 0.35$

$\frac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\frac{p_0}{\sigma_0} = 1$

$\frac{\sigma_1}{\sigma_0} = -1$

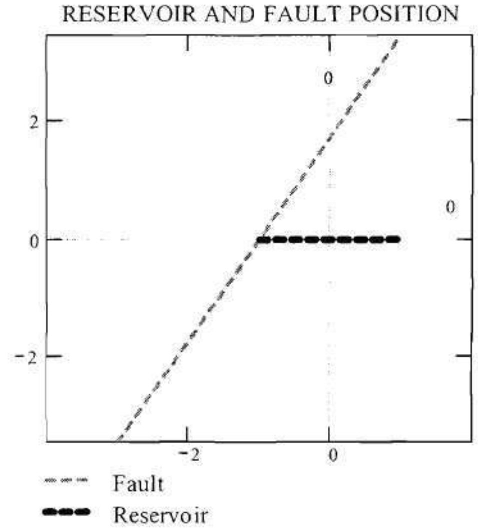
$\lambda = 0.75$

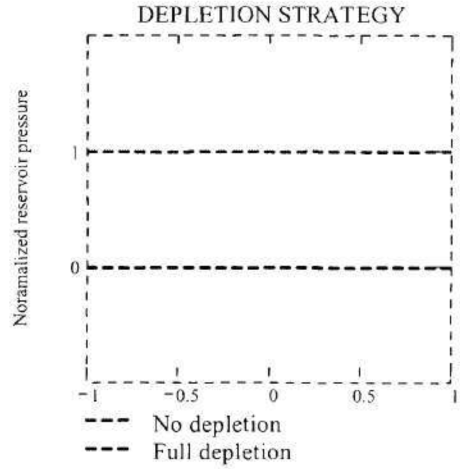
$\frac{p_f}{\sigma_0} = 0.3$

$\frac{2 \cdot a}{h_{max}} = 10$

$\gamma = 0 \cdot deg$

$\theta = 60 \cdot deg$

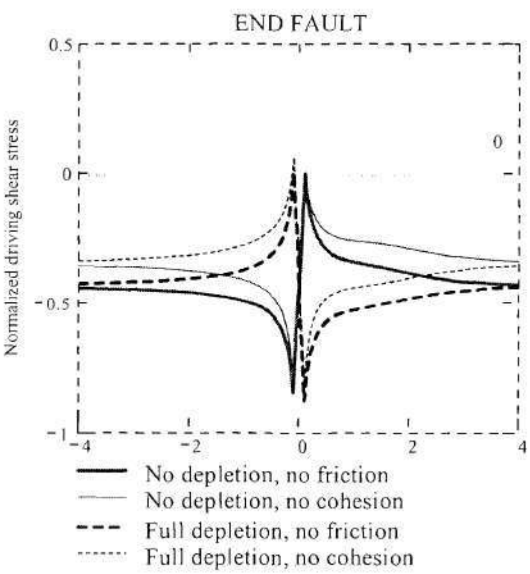




Cohesion $C_{1,2} = 18.89 \cdot MPa$, **no friction**

No cohesion, friction $\phi_{2,2} = 17.75 \cdot deg$

User defined cohesion $C_{3,2} = 9.6 \cdot MPa$ **and friction** $\phi_{3,2} = 9 \cdot deg$



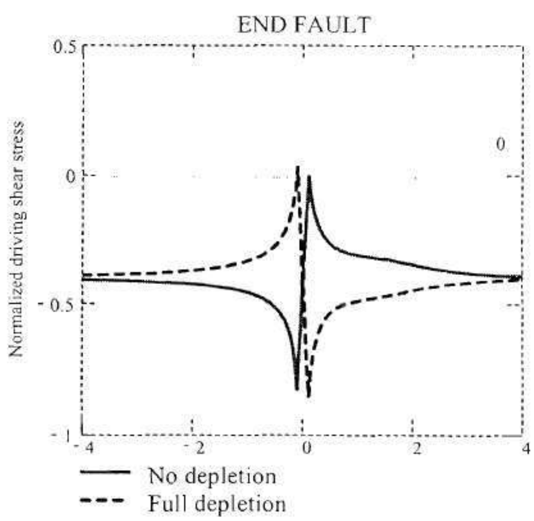


Figure 7.2.

NOTE: The figure on the left is plotted using the friction and cohesion computed in Sections 7.1 and 7.2 for the fault at the verge of instability. The figure on the right is plotted for the parameters identified by the user in Section 7.3.

Reference Parameters from Section 1:

$E = 1 \cdot 10^{10} \cdot Pa$

$\nu = 0.25$

$\nu_1 = 0.35$

$\frac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\frac{p_0}{\sigma_0} = 1$

$\frac{\sigma_1}{\sigma_0} = -1$

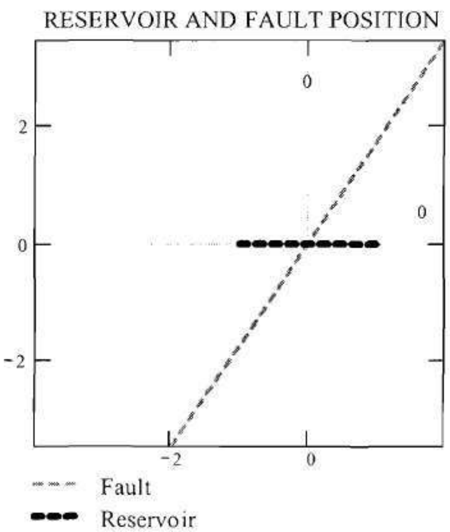
$\lambda = 0.75$

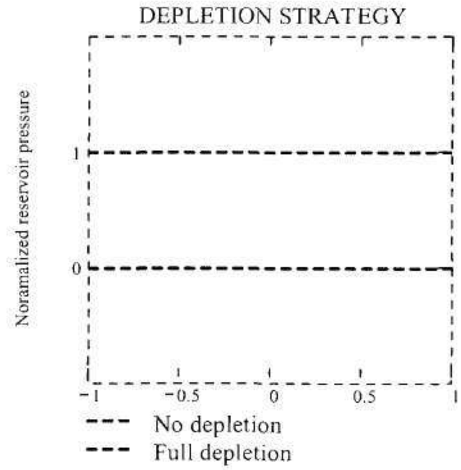
$\frac{p_f}{\sigma_0} = 0.3$

$\frac{2 \cdot a}{h_{max}} = 10$

$\gamma = 0 \cdot deg$

$\theta = 60 \cdot deg$

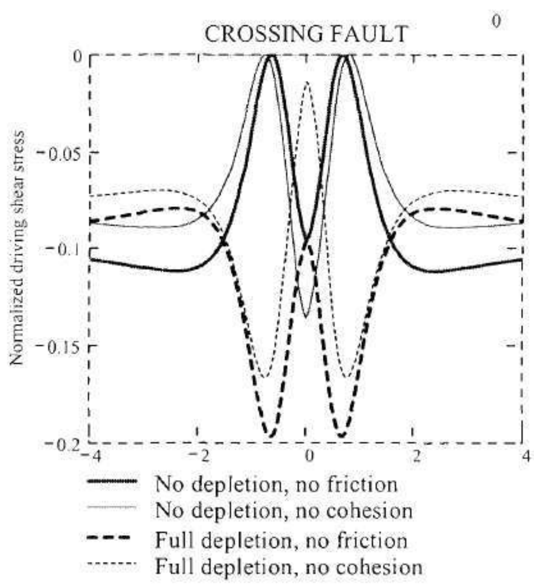




Cohesion $C_{1,3} = 11.98 \cdot MPa$, **no friction**

No cohesion, friction $\phi_{2,3} = 12.93 \cdot deg$

User defined cohesion $C_{3,3} = 5.9 \cdot MPa$ **and friction** $\phi_{3,3} = 6.5 \cdot deg$



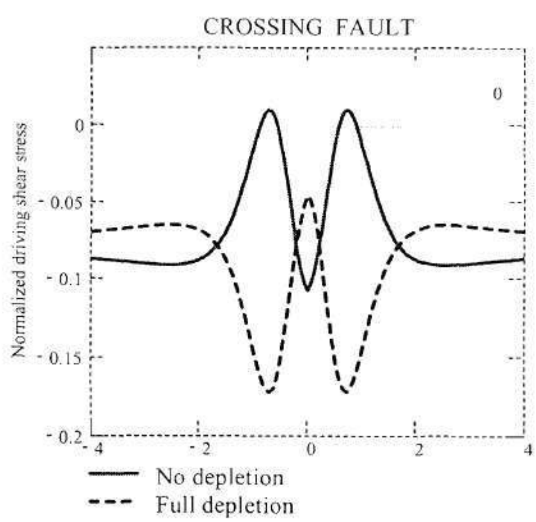


Figure 7.3.

NOTE: The figure on the left is plotted using the friction and cohesion computed in Sections 7.1 and 7.2 for the fault at the verge of instability. The figure on the right is plotted for the parameters identified by the user in Section 7.3.

8. Parametric Analysis of Depletion/Injection Strategy

As mentioned earlier, depletion/injection strategy is an important factor. Generally, changing the reservoir pressure does affect the driving shear stress distribution along the fault lines. The fault may not necessarily activate due to one depletion/injection strategy but may activate as a result of another one.

This part of Fault 99 is devised to demonstrate this effect and the same reference parameters as chosen in Section 1 are used for calculations in this section. Different depletion/injection strategies can be plotted and compared with respect to their potential impact on fault reactivation.

The driving shear stress distribution is calculated before depletion/injection such that it just touches the zero line (fault on the verge of instability). The reservoir is then partially depleted, and three intermediate stages are compared. After that, the reservoir is completely depleted ($p_0 = 0$ or $\Delta p = -p_0$). For comparison, all the driving shear stress distributions are shown in the same plot.

NOTE: Driving shear stress distributions for uniform depletion/injection was computed in Section 7. In this section, the user is provided with the option to compare uniform depletion/injection with three other pressure distributions. The plots below are grouped in two sections, 8.1 and 8.2, representing two sets of examples of pressure distributions along the reservoir. Other cases can be easily considered by disabling the regions with suggested pressure distributions and introducing the desired ones. The same parameters as in Sections 1 and 7 are used in this section. They are shown below only for reference and cannot be changed at this point.

$E = 1 \cdot 10^{10} \cdot Pa$

$E_1 = 0.3333 \cdot E$

$\nu = 0.25$

$\nu_1 = 0.35$

$a = 150\ m$

$h_{max} = 30\ m$

$\sigma_1 = -100 \cdot MPa$

$\lambda = 0.75$

$\sigma_3(\lambda, \sigma_1) = -75 \cdot MPa$

$p_f = 30 \cdot MPa$

$\gamma = 0 \cdot deg$

$\theta = 60 \cdot deg$

$\alpha_p = 1$

$p_0 = 100 \cdot MPa$

8.1. Example of Uniform, Linear, and Step-Like Intermediate Depletions

Reservoir is *linearly* depleted from one end to the other:

$f_1 := 1$ $f_2 := 0$ $\Delta p_{max} := -p_0$

$\xi_{min} := -1$ $\xi_{max} := 1$

$$f_{\Delta p}(\xi) := f_1 + \frac{\xi - \xi_{min}}{\xi_{max} - \xi_{min}} \cdot (f_2 - f_1)$$

$$f_{\Delta p\_L}(\xi) := f_1 + \frac{\xi - \xi_{min}}{\xi_{max} - \xi_{min}} \cdot (f_2 - f_1)$$

NOTE: These two expressions should be disabled and replaced by the desired pressure distribution if needed. The second formula is used only for the convenience of plotting.

The following calculations are for evaluating the coefficients α and β:

$j := 1, 2 .. N$

$R_{\alpha 1_j} := f_{\Delta p}(\xi_j)$

$R_{\alpha 2_j} := 1$

$$R_\alpha(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := R_{\alpha 1} \cdot \frac{\alpha_p \cdot \Delta p_{max}}{\sigma_0} \ldots \\ + R_{\alpha 2} \cdot \frac{1 - \varepsilon_1(E, E_1, \nu, \nu_1) \cdot \sigma_{yy\_remote}(\gamma, \lambda, \sigma_1) \ldots + (-1) \cdot \varepsilon_k(E, E_1, \nu, \nu_1) \cdot \sigma_{xx\_remote}(\gamma, \lambda, \sigma_1) + \alpha_p \cdot p_0}{\sigma_0}$$

$$\varepsilon_\alpha(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \text{lsolve}(A(E, E_1, \nu, \nu_1, h_{max}, a), R_\alpha(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p))$$

$$\alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \varepsilon_\alpha(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) \cdot \sigma_0$$

$j := 1, 2 .. \text{last}(\xi_f)$

$$\alpha := \alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$$

$$\beta := \beta'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$$

Coulomb shear stress and strength along the fault lines as a result of linear depletion are:

$\tau_{Clmb\_no\_friction\_L_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction_j}, p_f\right)$

$\tau_{Clmb\_no\_friction\_end\_L_i} := \tau_{Coulomb}\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction\_end}, p_f\right)$

$\tau_{no\_friction\_L_{i1,j}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction_j}, C_{no\_friction_j}, p_f\right)$

$\tau_{no\_friction\_end\_L_i} := \tau_f\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction\_end}, C_{no\_friction\_end}, p_f\right)$

$\tau_{Clmb\_no\_cohesion\_L_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion_j}, p_f\right)$

$\tau_{Clmb\_no\_cohesion\_end\_L_i} := \tau_{Coulomb}\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion\_end}, p_f\right)$

$\tau_{no\_cohesion\_L_{i1,j}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion_j}, C_{no\_cohesion_j}, p_f\right)$

$\tau_{no\_cohesion\_end\_L_i} := \tau_f\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion\_end}, C_{no\_cohesion\_end}, p_f\right)$

$\tau_{Clmb\_arbitrary\_L_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_j}, p_f\right)$

$\tau_{Clmb\_arbitrary\_end\_L_i} := \tau_{Coulomb}\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary\_end}, p_f\right)$

$\tau_{arbitrary\_L_{i1,j}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_j}, C_{arbitrary_j}, p_f\right)$

$\tau_{arbitrary\_end\_L_i} := \tau_f\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary\_end}, C_{arbitrary\_end}, p_f\right)$

Reservoir is depleted by a *step-like* distribution of pressure at the center from the left:

$$f_{\Delta p}(\xi) := \begin{vmatrix} 1 & \text{if } \xi \le 0 \\ 0 & \text{otherwise} \end{vmatrix}$$

$$f_{\Delta p\_2}(\xi) := \begin{vmatrix} 0.98 & \text{if } \xi \le 0 \\ 0.02 & \text{otherwise} \end{vmatrix}$$

NOTE: These two expressions should be disabled and replaced by the desired pressure distribution if needed. The second formula is used only for the convenience of plotting.

The following calculations are for evaluating the coefficients α and β:

$j := 1, 2..N$

$R_{\alpha 1_j} := f_{\Delta p}\left(\xi_j\right)$

$R_{\alpha2_j} := 1$

$$R_{\alpha}(E,E_1,\nu,\nu_1,\gamma,\lambda,\sigma_1,\Delta p_{max},p_0,\alpha_p) := R_{\alpha1}\cdot\frac{\alpha_p\cdot\Delta p_{max}}{\sigma_0}\ldots + R_{\alpha2}\cdot\frac{(1-\varepsilon_1(E,E_1,\nu,\nu_1))\cdot\sigma_{yy\_remote}(\gamma,\lambda,\sigma_1)\ldots + (-1)\cdot\varepsilon_k(E,E_1,\nu,\nu_1)\cdot\sigma_{xx\_remote}(\gamma,\lambda,\sigma_1) + \alpha_p\cdot p_0}{\sigma_0}$$

$$\varepsilon_{\alpha}(E,E_1,\nu,\nu_1,h_{max},a,\gamma,\lambda,\sigma_1,\Delta p_{max},p_0,\alpha_p) := \mathrm{lsolve}(A(E,E_1,\nu,\nu_1,h_{max},a),R_{\alpha}(E,E_1,\nu,\nu_1,\gamma,\lambda,\sigma_1,\Delta p_{max},p_0,\alpha_p))$$

$$\alpha'(E,E_1,\nu,\nu_1,h_{max},a,\gamma,\lambda,\sigma_1,\Delta p_{max},p_0,\alpha_p) := \varepsilon_{\alpha}(E,E_1,\nu,\nu_1,h_{max},a,\gamma,\lambda,\sigma_1,\Delta p_{max},p_0,\alpha_p)\cdot\sigma_0$$

$j := 1,2..\mathrm{last}(\xi_f)$

$\alpha := \alpha'(E,E_1,\nu,\nu_1,h_{max},a,\gamma,\lambda,\sigma_1,\Delta p_{max},p_0,\alpha_p)$

$\beta := \beta'(E,E_1,\nu,\nu_1,h_{max},a,\gamma,\lambda,\sigma_1,\Delta p_{max},p_0,\alpha_p)$

Coulomb shear stress and strength along the fault lines as a result of step-like distribution of pressure depletion at the left half of the reservoir are:

$\tau_{Clmb\_no\_friction\_step_{i1,j}} := \tau_{Coulomb}(\zeta_{i1,j},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{no\_friction_j},p_f)$

$\tau_{Clmb\_no\_friction\_end\_step_i} := \tau_{Coulomb}(\zeta_{end_i},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{no\_friction\_end},p_f)$

$\tau_{no\_friction\_step_{i1,j}} := \tau_f(\zeta_{i1,j},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{no\_friction_j},C_{no\_friction_j},p_f)$

$\tau_{no\_friction\_end\_step_i} := \tau_f(\zeta_{end_i},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{no\_friction\_end},C_{no\_friction\_end},p_f)$

$\tau_{Clmb\_no\_cohesion\_step_{i1,j}} := \tau_{Coulomb}(\zeta_{i1,j},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{no\_cohesion_j},p_f)$

$\tau_{Clmb\_no\_cohesion\_end\_step_i} := \tau_{Coulomb}(\zeta_{end_i},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{no\_cohesion\_end},p_f)$

$\tau_{no\_cohesion\_step_{i1,j}} := \tau_f(\zeta_{i1,j},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{no\_cohesion_j},C_{no\_cohesion_j},p_f)$

$\tau_{no\_cohesion\_end\_step_i} := \tau_f(\zeta_{end_i},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{no\_cohesion\_end},C_{no\_cohesion\_end},p_f)$

$$\tau_{Clmb\_arbitrary\_step_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_j}, p_f\right)$$

$$\tau_{Clmb\_arbitrary\_end\_step_i} := \tau_{Coulomb}\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary\_end}, p_f\right)$$

$$\tau_{arbitrary\_step_{i1,j}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_j}, C_{arbitrary_j}, p_f\right)$$

$$\tau_{arbitrary\_end\_step_i} := \tau_f\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary\_end}, C_{arbitrary\_end}, p_f\right)$$

Reference Parameters from Section 1:

$f_{\Delta p\_intermediate} := 0.5$ Intermediate pressure distribution in the reservoir (needed only for plotting)

$E = 1 \cdot 10^{10} \cdot Pa$

$\nu = 0.25$

$\nu_1 = 0.35$

$\frac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\frac{p_0}{\sigma_0} = 1$

$\frac{\sigma_1}{\sigma_0} = -1$

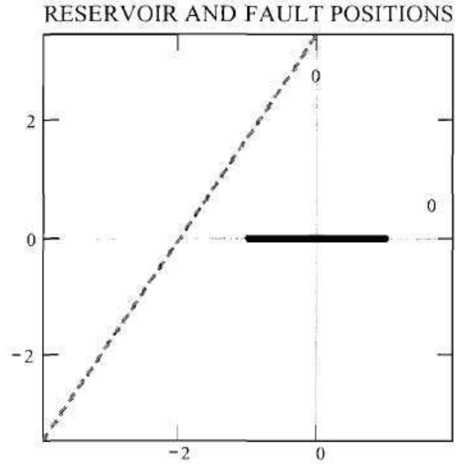
$\lambda = 0.75$

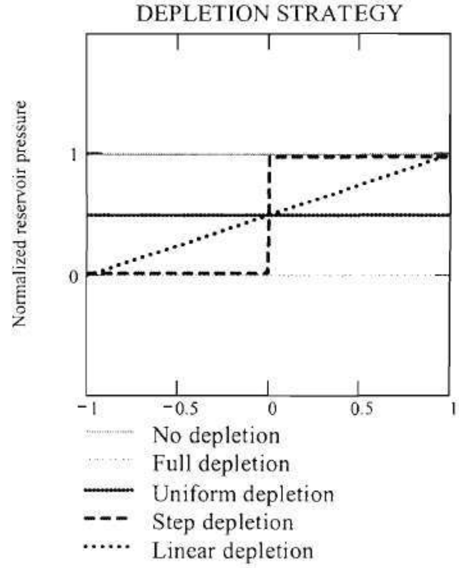
$\frac{p_f}{\sigma_0} = 0.3$

$\frac{2 \cdot a}{h_{max}} = 10$

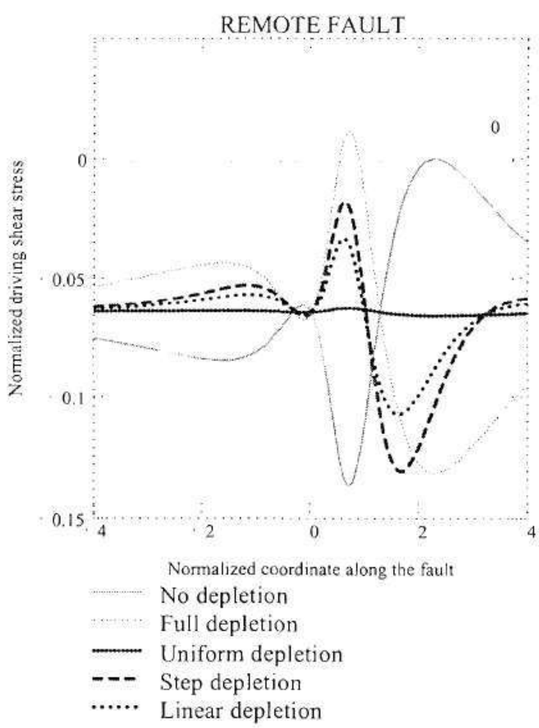
$\gamma = 0 \cdot deg$

$\theta = 60 \cdot deg$





Cohesion $C_{1,1} = 11.57 \cdot MPa$**, no friction**



No cohesion, friction $\phi_{2,1} = 12.56 \cdot deg$

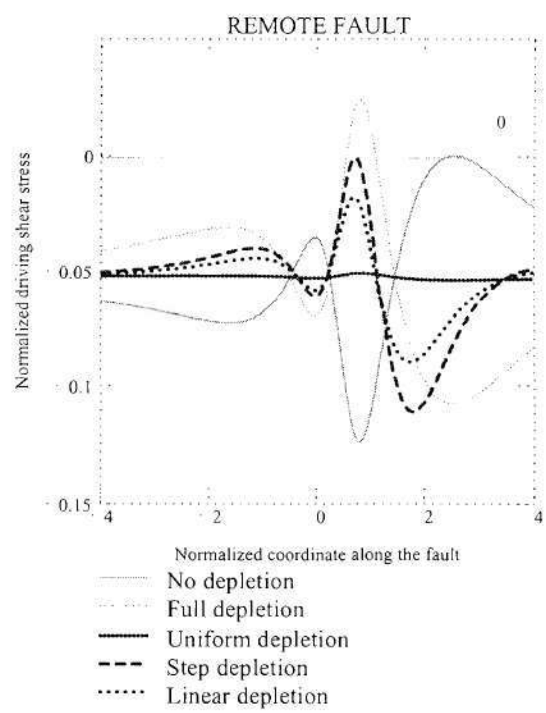


Figure 8.1.

NOTE: The figures above are plotted using the friction and cohesion computed in Sections 7.1 and 7.2 for the fault at the verge of instability.

Reference Parameters from Section 1:

$E = 1 \cdot 10^{10} \cdot Pa$

$\nu = 0.25$

$\nu_1 = 0.35$

$\frac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\frac{p_0}{\sigma_0} = 1$

$\frac{\sigma_1}{\sigma_0} = 1$

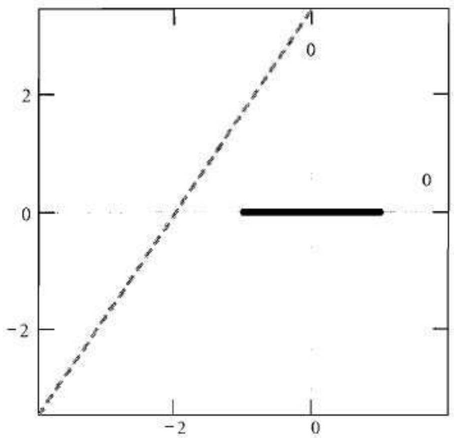
$\lambda = 0.75$

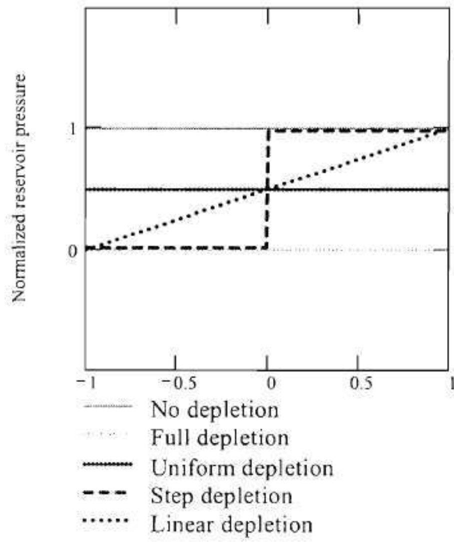
$\frac{p_f}{\sigma_0} = 0.3$

$\frac{2 \cdot a}{h_{max}} = 10$

$\gamma = 0 \cdot deg$

$\theta = 60 \cdot deg$





User defined cohesion $C_{3,1} = 5.8 \cdot MPa$
and friction $\phi_{31} = 6.3 \cdot deg$

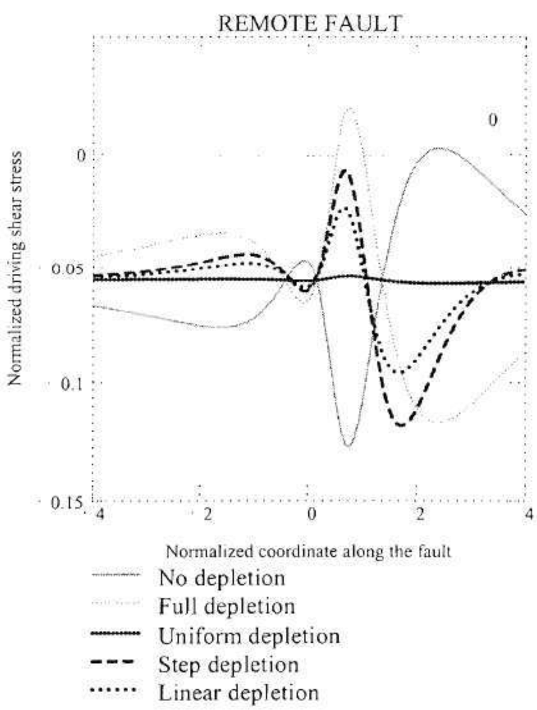


Figure 8.2.

NOTE: The figure above is plotted using the friction and cohesion identified by the user in Section 7.3.

Reference
Parameters
from Section 1:

$E = 1 \cdot 10^{10} \cdot \mathrm{Pa}$

$\nu = 0.25$

$\nu_1 = 0.35$

$\frac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\frac{p_0}{\sigma_0} = 1$

$\frac{\sigma_1}{\sigma_0} = 1$

$\lambda = 0.75$

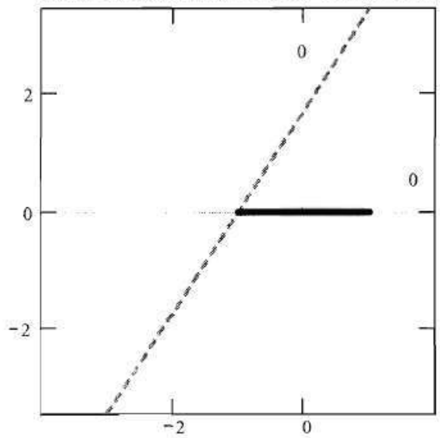
$\frac{p_f}{\sigma_0} = 0.3$

$\frac{2 \cdot a}{h_{max}} = 10$

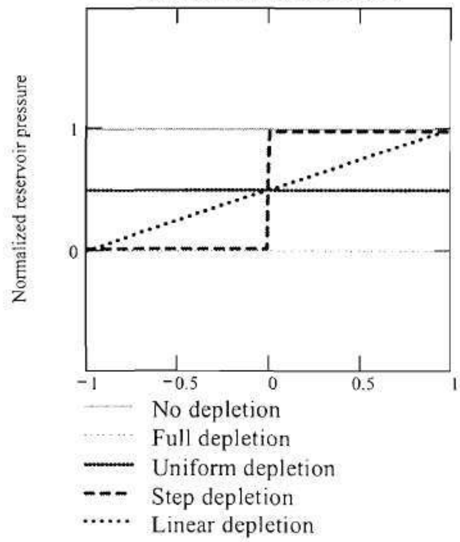
$\gamma = 0 \cdot \mathrm{deg}$

$\theta = 60 \cdot \mathrm{deg}$

RESERVOIR AND FAULT POSITIONS

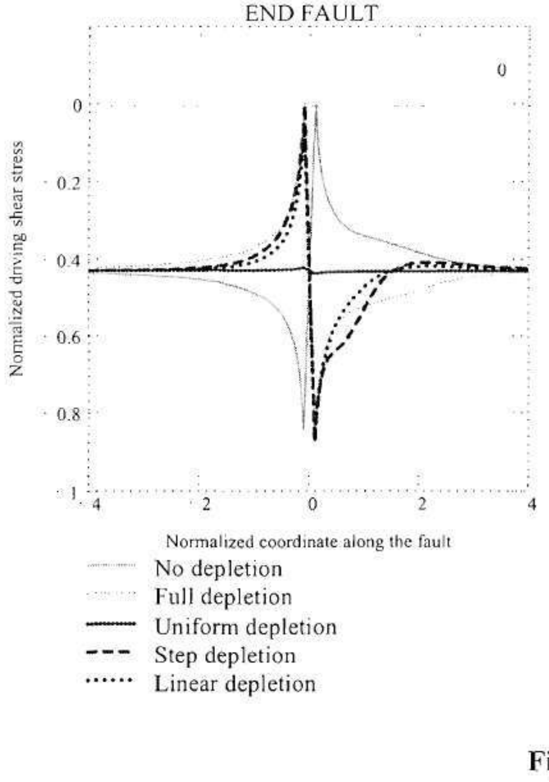


DEPLETION STRATEGY



Cohesion $C_{1,2} = 18.89 \cdot \mathrm{MPa}$, **no friction**

END FAULT



No cohesion, friction $\phi_{2,2} = 17.75 \cdot \mathrm{deg}$

END FAULT

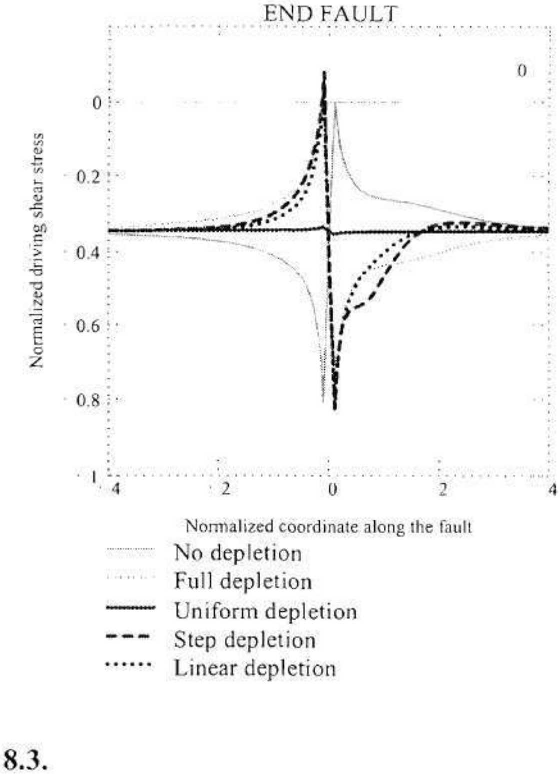


Figure 8.3.

NOTE: The figures above are plotted using the friction and cohesion computed in Sections 7.1 and 7.2 for the fault at the verge of instability.

Reference Parameters from Section 1:

$E = 1 \cdot 10^{10} \cdot \mathrm{Pa}$

$\nu = 0.25$

$\nu_1 = 0.35$

$\frac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\frac{p_0}{\sigma_0} = 1$

$\frac{\sigma_1}{\sigma_0} = -1$

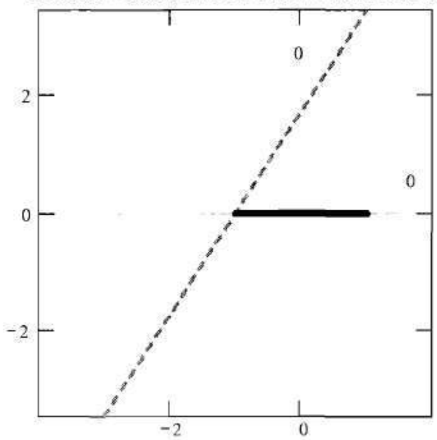
$\lambda = 0.75$

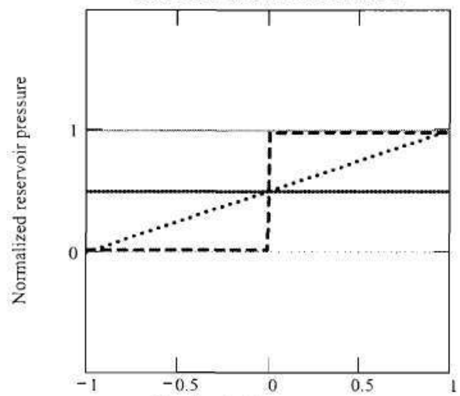
$\frac{p_f}{\sigma_0} = 0.3$

$\frac{2 \cdot a}{h_{max}} = 10$

$\gamma = 0 \cdot \mathrm{deg}$

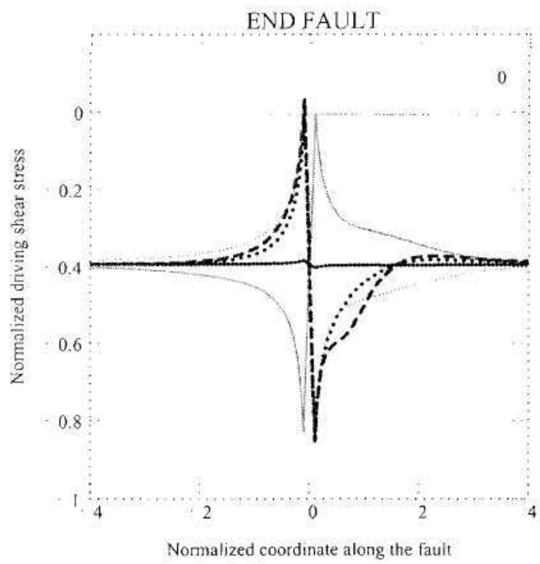
$\theta = 60 \cdot \mathrm{deg}$





No depletion
Full depletion
Uniform depletion
Step depletion
Linear depletion

User defined cohesion $C_{3,2} = 9.6 \cdot \mathrm{MPa}$
and friction $\phi_{3,2} = 9 \cdot \mathrm{deg}$



No depletion
Full depletion
Uniform depletion
Step depletion
Linear depletion

Figure 8.4.

NOTE: The figure above is plotted using the friction and cohesion identified by the user in Section 7.3.

Reference Parameters from Section 1:

$E = 1 \cdot 10^{10} \cdot \mathrm{Pa}$

$\nu = 0.25$

$\nu_1 = 0.35$

$\frac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\frac{p_0}{\sigma_0} = 1$

$\frac{\sigma_1}{\sigma_0} = 1$

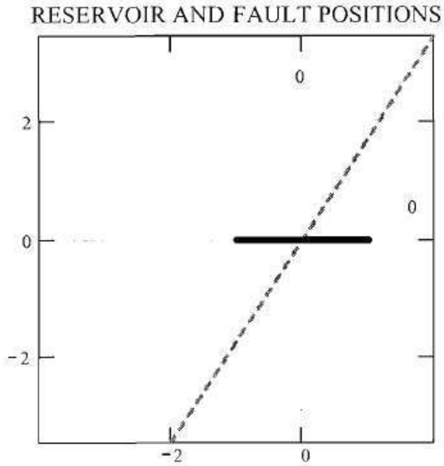
$\lambda = 0.75$

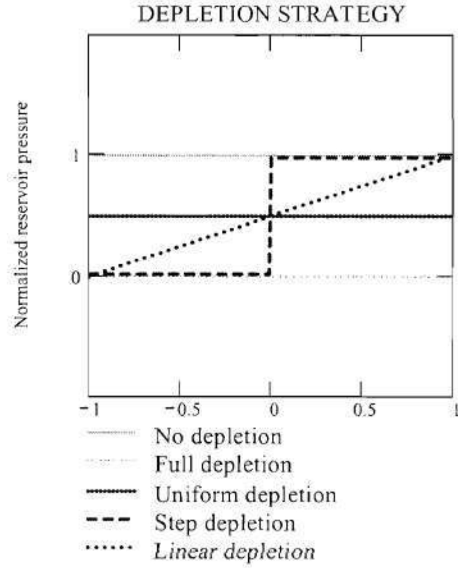
$\frac{p_f}{\sigma_0} = 0.3$

$\frac{2 \cdot a}{h_{max}} = 10$

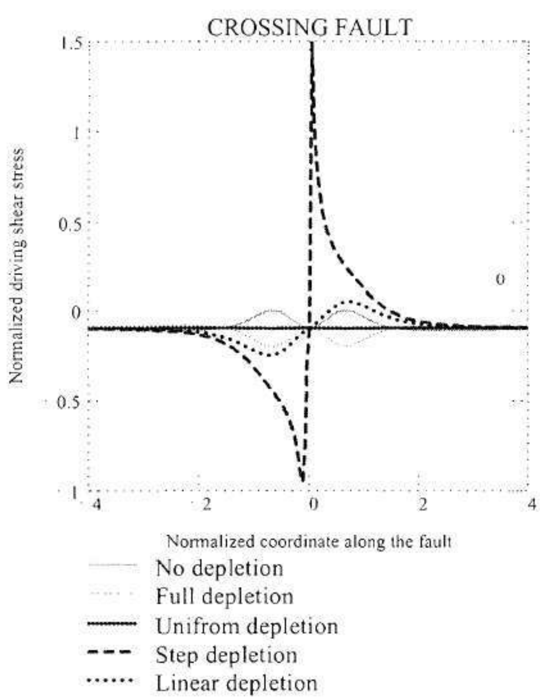
$\gamma = 0 \cdot \mathrm{deg}$

$\theta = 60 \cdot \mathrm{deg}$





Cohesion $C_{1,3} = 11.98 \cdot \mathrm{MPa}$, **no friction**



No cohesion, friction $\phi_{2,3} = 12.93 \cdot \mathrm{deg}$

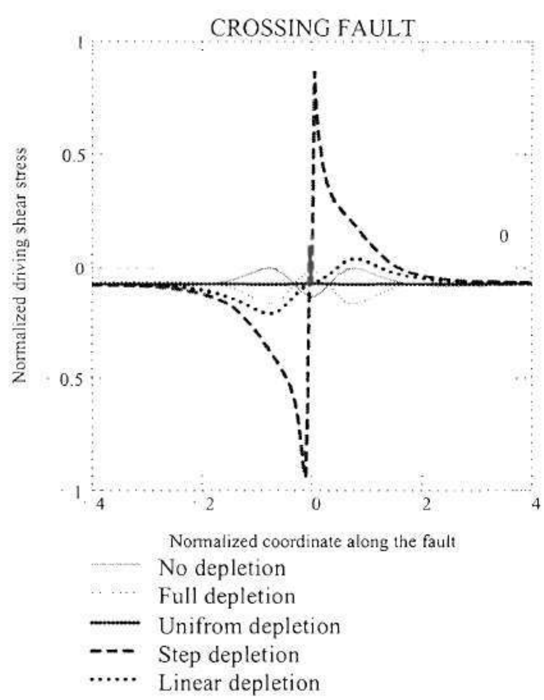


Figure 8.5.

NOTE: The figures above are plotted using the friction and cohesion computed in Sections 7.1 and 7.2 for the fault at the verge of instability.

Reference Parameters from Section 1:

$E = 1 \cdot 10^{10} \cdot Pa$

$\nu = 0.25$

$\nu_1 = 0.35$

$\frac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\frac{p_0}{\sigma_0} = 1$

$\frac{\sigma_1}{\sigma_0} = 1$

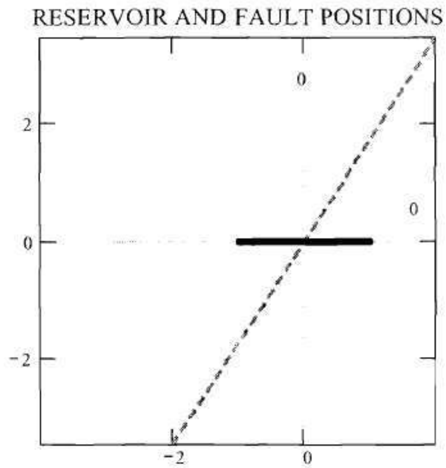
$\lambda = 0.75$

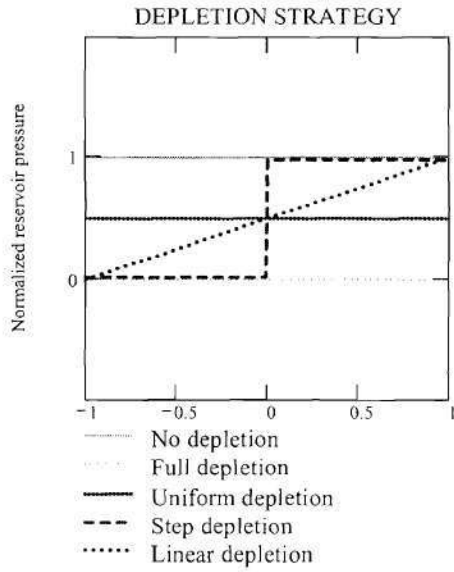
$\frac{p_f}{\sigma_0} = 0.3$

$\frac{2 \cdot a}{h_{max}} = 10$

$\gamma = 0 \cdot deg$

$\theta = 60 \cdot deg$





User defined cohesion $C_{3,3} = 5.9 \cdot MPa$
and friction $\phi_{3,3} = 6.5 \cdot deg$

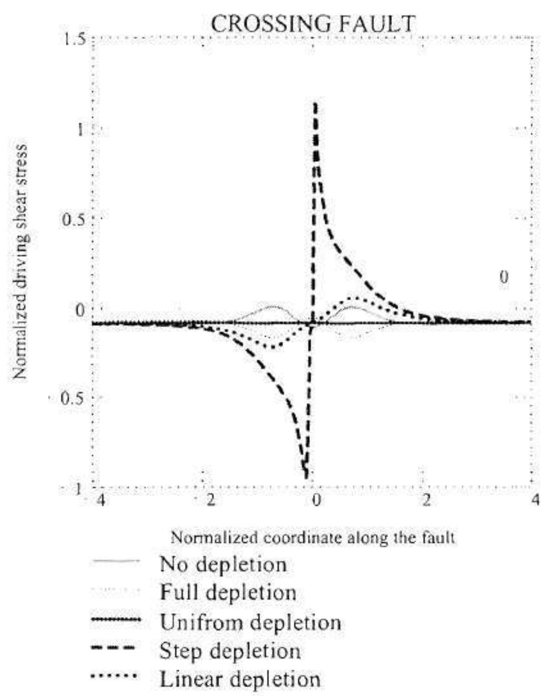


Figure 8.6.

NOTE: The figure above is plotted using the friction and cohesion identified by the user in Section 7.3.

8.2. Example of Three Step-Like Intermediate Depletions

Reservoir is depleted by a *step-like* distribution of pressure at the right quarter:

$$f_{\Delta p}(\xi) := \begin{cases} 1 & \text{if } \xi \ge 0.5 \\ 0 & \text{otherwise} \end{cases}$$

NOTE: These two expressions should be disabled and replaced by the desired pressure distribution if needed. The second formula is used only for the convenience of plotting.

$$f_{\Delta p\_1}(\xi) := \begin{cases} 0.98 & \text{if } \xi \ge 0.5 \\ 0.02 & \text{otherwise} \end{cases}$$

The following calculations are for evaluating the coefficients α and β:

$j := 1, 2 .. N$

$R_{\alpha 1_j} := f_{\Delta p}(\xi_j)$

$R_{\alpha 2_j} := 1$

$$R_{\alpha}(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := R_{\alpha 1} \cdot \frac{\alpha_p \cdot \Delta p_{max}}{\sigma_0} \ldots \\ + R_{\alpha 2} \cdot \frac{\begin{array}{c} 1 - \varepsilon_1(E, E_1, \nu, \nu_1) \cdot \sigma_{yy\_remote}(\gamma, \lambda, \sigma_1) \ldots \\ + (-1) \cdot \varepsilon_k(E, E_1, \nu, \nu_1) \cdot \sigma_{xx\_remote}(\gamma, \lambda, \sigma_1) + \alpha_p \cdot p_0 \end{array}}{\sigma_0}$$

$$\varepsilon_{\alpha}(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \text{lsolve}(A(E, E_1, \nu, \nu_1, h_{max}, a), R_{\alpha}(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p))$$

$$\alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \varepsilon_{\alpha}(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) \cdot \sigma_0$$

$j := 1, 2 .. \text{last}(\xi_f)$

$\alpha := \alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$

$\beta := \beta'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$

Coulomb shear stress and strength along the fault lines as a result of step-like distribution of pressure depletion at the right quarter of the reservoir are:

$\tau_{Clmb\_no\_friction\_rev\_step\_1_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction_j}, p_f\right)$

$\tau_{Clmb\_no\_friction\_end\_rev\_step\_1_i} := \tau_{Coulomb}\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction\_end}, p_f\right)$

$\tau_{no\_friction\_rev\_step\_1_{i1,j}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction_j}, C_{no\_friction_j}, p_f\right)$

$\tau_{no\_friction\_end\_rev\_step\_1_i} := \tau_f\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction\_end}, C_{no\_friction\_end}, p_f\right)$

$\tau_{Clmb\_no\_cohesion\_rev\_step\_1_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion_j}, p_f\right)$

$\tau_{Clmb\_no\_cohesion\_end\_rev\_step\_1_i} := \tau_{Coulomb}\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion\_end}, p_f\right)$

$\tau_{no\_cohesion\_rev\_step\_1_{i1,j}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion_j}, C_{no\_cohesion_j}, p_f\right)$

$\tau_{no\_cohesion\_end\_rev\_step\_1_i} := \tau_f\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion\_end}, C_{no\_cohesion\_end}, p_f\right)$

$\tau_{Clmb\_arbitrary\_rev\_step\_1_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_j}, p_f\right)$

$\tau_{Clmb\_arbitrary\_end\_rev\_step\_1_i} := \tau_{Coulomb}\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary\_end}, p_f\right)$

$\tau_{arbitrary\_rev\_step\_1_{i1,j}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_j}, C_{arbitrary_j}, p_f\right)$

$\tau_{arbitrary\_end\_rev\_step\_1_i} := \tau_f\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary\_end}, C_{arbitrary\_end}, p_f\right)$

Reservoir is depleted by a step-like distribution of pressure at the center from the right:

$f_{\Delta p}(\xi) := \begin{cases} 1 & \text{if } \xi \ge 0 \\ 0 & \text{otherwise} \end{cases}$

NOTE: These two expressions should be disabled and replaced by the desired pressure distribution if needed. The second formula is used only for the convenience of plotting.

$f_{\Delta p\_2}(\xi) := \begin{cases} 0.98 & \text{if } \xi \ge 0 \\ 0.02 & \text{otherwise} \end{cases}$

The following calculations are for evaluating the coefficients α and β:

$j := 1, 2 .. N$

$R_{\alpha 1_j} := f_{\Delta p}\left(\xi_j\right)$

$R_{\alpha 2_j} := 1$

$$R_{\alpha}\left(E,E_1,\nu,\nu_1,\gamma,\lambda,\sigma_1,\Delta p_{max},p_0,\alpha_p\right) := R_{\alpha 1}\cdot\frac{\alpha_p\cdot\Delta p_{max}}{\sigma_0} \ldots + R_{\alpha 2}\cdot\frac{\left(1-\varepsilon_1\left(E,E_1,\nu,\nu_1\right)\right)\cdot\sigma_{yy\_remote}\left(\gamma,\lambda,\sigma_1\right) \ldots + (-1)\cdot\varepsilon_k\left(E,E_1,\nu,\nu_1\right)\cdot\sigma_{xx\_remote}\left(\gamma,\lambda,\sigma_1\right) + \alpha_p\cdot p_0}{\sigma_0}$$

$$\varepsilon_{\alpha}\left(E,E_1,\nu,\nu_1,h_{max},a,\gamma,\lambda,\sigma_1,\Delta p_{max},p_0,\alpha_p\right) := \text{lsolve}\left(A\left(E,E_1,\nu,\nu_1,h_{max},a\right),R_{\alpha}\left(E,E_1,\nu,\nu_1,\gamma,\lambda,\sigma_1,\Delta p_{max},p_0,\alpha_p\right)\right)$$

$$\alpha'\left(E,E_1,\nu,\nu_1,h_{max},a,\gamma,\lambda,\sigma_1,\Delta p_{max},p_0,\alpha_p\right) := \varepsilon_{\alpha}\left(E,E_1,\nu,\nu_1,h_{max},a,\gamma,\lambda,\sigma_1,\Delta p_{max},p_0,\alpha_p\right)\cdot\sigma_0$$

$$j := 1,2..\text{last}\left(\xi_f\right)$$

$$\alpha := \alpha'\left(E,E_1,\nu,\nu_1,h_{max},a,\gamma,\lambda,\sigma_1,\Delta p_{max},p_0,\alpha_p\right)$$

$$\beta := \beta'\left(E,E_1,\nu,\nu_1,h_{max},a,\gamma,\lambda,\sigma_1,\Delta p_{max},p_0,\alpha_p\right)$$

Coulomb shear stress and strength along the fault lines as a result of step-like distribution of pressure depletion at the right half of the reservoir are:

$$\tau_{Clmb\_no\_friction\_rev\_step\_2_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{no\_friction_j},p_f\right)$$

$$\tau_{Clmb\_no\_friction\_end\_rev\_step\_2_i} := \tau_{Coulomb}\left(\zeta_{end_i},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{no\_friction\_end},p_f\right)$$

$$\tau_{no\_friction\_rev\_step\_2_{i1,j}} := \tau_f\left(\zeta_{i1,j},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{no\_friction_j},C_{no\_friction_j},p_f\right)$$

$$\tau_{no\_friction\_end\_rev\_step\_2_i} := \tau_f\left(\zeta_{end_i},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{no\_friction\_end},C_{no\_friction\_end},p_f\right)$$

$$\tau_{Clmb\_no\_cohesion\_rev\_step\_2_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{no\_cohesion_j},p_f\right)$$

$$\tau_{Clmb\_no\_cohesion\_end\_rev\_step\_2_i} := \tau_{Coulomb}\left(\zeta_{end_i},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{no\_cohesion\_end},p_f\right)$$

$$\tau_{no\_cohesion\_rev\_step\_2_{i1,j}} := \tau_f\left(\zeta_{i1,j},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{no\_cohesion_j},C_{no\_cohesion_j},p_f\right)$$

$$\tau_{no\_cohesion\_end\_rev\_step\_2_i} := \tau_f\left(\zeta_{end_i},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{no\_cohesion\_end},C_{no\_cohesion\_end},p_f\right)$$

$$\tau_{Clmb\_arbitrary\_rev\_step\_2_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{arbitrary_j},p_f\right)$$

$$\tau_{Clmb\_arbitrary\_end\_rev\_step\_2_i} := \tau_{Coulomb}\left(\zeta_{end_i},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{arbitrary\_end},p_f\right)$$

$$\tau_{arbitrary\_rev\_step\_2_{i1,j}} := \tau_f\left(\zeta_{i1,j},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{arbitrary_j},C_{arbitrary_j},p_f\right)$$

$$\tau_{arbitrary\_end\_rev\_step\_2_i} := \tau_f\left(\zeta_{end_i},\theta,\gamma,\lambda,\sigma_1,\alpha,\beta,\phi_{arbitrary\_end},C_{arbitrary\_end},p_f\right)$$

Reservoir is depleted by a step-like distribution of pressure at the right three quarters:

$$f_{\Delta p}(\xi) := \begin{cases} 1 & \text{if } \xi \ge -0.5 \\ 0 & \text{otherwise} \end{cases}$$

NOTE: These two expressions should be disabled and replaced by the desired pressure distribution if needed. The second formula is used only for the convenience of plotting.

$$f_{\Delta p\_3}(\xi) := \begin{cases} 0.98 & \text{if } \xi \ge -0.5 \\ 0.02 & \text{otherwise} \end{cases}$$

The following calculations are for evaluating the coefficients α and β:

$j := 1, 2 .. N$

$R_{\alpha 1_j} := f_{\Delta p}(\xi_j)$

$R_{\alpha 2_j} := 1$

$$R_{\alpha}(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := R_{\alpha 1} \cdot \frac{\alpha_p \cdot \Delta p_{max}}{\sigma_0} \dots + R_{\alpha 2} \cdot \frac{\left(1 - \varepsilon_1(E, E_1, \nu, \nu_1)\right) \cdot \sigma_{yy\_remote}(\gamma, \lambda, \sigma_1) \dots + (-1) \cdot \varepsilon_k(E, E_1, \nu, \nu_1) \cdot \sigma_{xx\_remote}(\gamma, \lambda, \sigma_1) + \alpha_p \cdot p_0}{\sigma_0}$$

$\varepsilon_{\alpha}(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \text{lsolve}\left(A(E, E_1, \nu, \nu_1, h_{max}, a), R_{\alpha}(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)\right)$

$\alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \varepsilon_{\alpha}(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) \cdot \sigma_0$

$j := 1, 2 .. \text{last}(\xi_f)$

$\alpha := \alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$

$\beta := \beta'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$

Coulomb shear stress and strength along the fault lines as a result of step-like distribution of pressure depletion at the right three quarters of the reservoir are:

$\tau_{Clmb\_no\_friction\_rev\_step\_3_{i1,j}} := \tau_{Coulomb}(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction_j}, p_f)$

$\tau_{Clmb\_no\_friction\_end\_rev\_step\_3_i} := \tau_{Coulomb}(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction\_end}, p_f)$

$\tau_{no\_friction\_rev\_step\_3_{i1,j}} := \tau_f(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction_j}, C_{no\_friction_j}, p_f)$

$\tau_{no\_friction\_end\_rev\_step\_3_i} := \tau_f(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_friction\_end}, C_{no\_friction\_end}, p_f)$

$$\tau_{Clmb\_no\_cohesion\_rev\_step\_3_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion_j}, p_f\right)$$

$$\tau_{Clmb\_no\_cohesion\_end\_rev\_step\_3_i} := \tau_{Coulomb}\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion\_end}, p_f\right)$$

$$\tau_{no\_cohesion\_rev\_step\_3_{i1,j}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion_j}, C_{no\_cohesion_j}, p_f\right)$$

$$\tau_{no\_cohesion\_end\_rev\_step\_3_i} := \tau_f\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{no\_cohesion\_end}, C_{no\_cohesion\_end}, p_f\right)$$

$$\tau_{Clmb\_arbitrary\_rev\_step\_3_{i1,j}} := \tau_{Coulomb}\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_j}, p_f\right)$$

$$\tau_{Clmb\_arbitrary\_end\_rev\_step\_3_i} := \tau_{Coulomb}\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary\_end}, p_f\right)$$

$$\tau_{arbitrary\_rev\_step\_3_{i1,j}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_j}, C_{arbitrary_j}, p_f\right)$$

$$\tau_{arbitrary\_end\_rev\_step\_3_i} := \tau_f\left(\zeta_{end_i}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary\_end}, C_{arbitrary\_end}, p_f\right)$$

Reference Parameters from Section 1:

$E = 1 \cdot 10^{10} \cdot Pa$

$\nu = 0.25$

$\nu_1 = 0.35$

$\frac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\frac{p_0}{\sigma_0} = 1$

$\frac{\sigma_1}{\sigma_0} = 1$

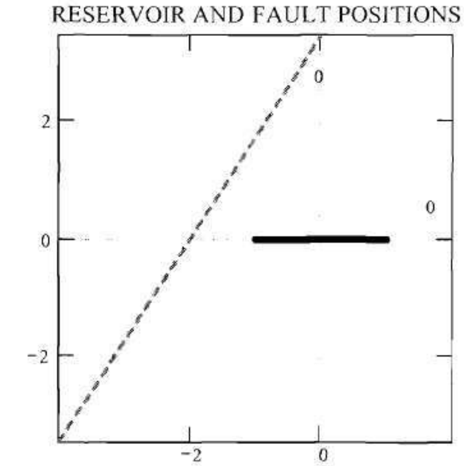
$\lambda = 0.75$

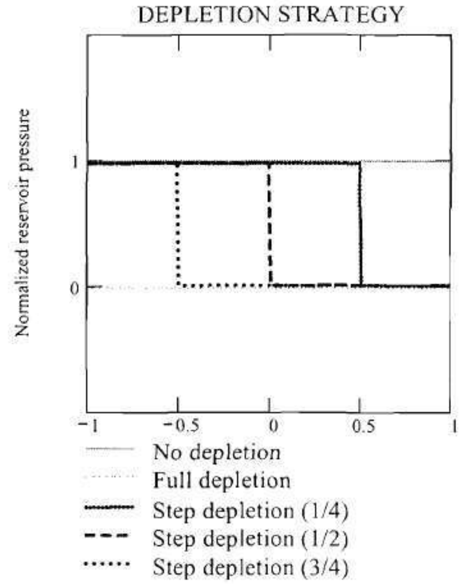
$\frac{p_f}{\sigma_0} = 0.3$

$\frac{2 \cdot a}{h_{max}} = 10$

$\gamma = 0 \cdot deg$

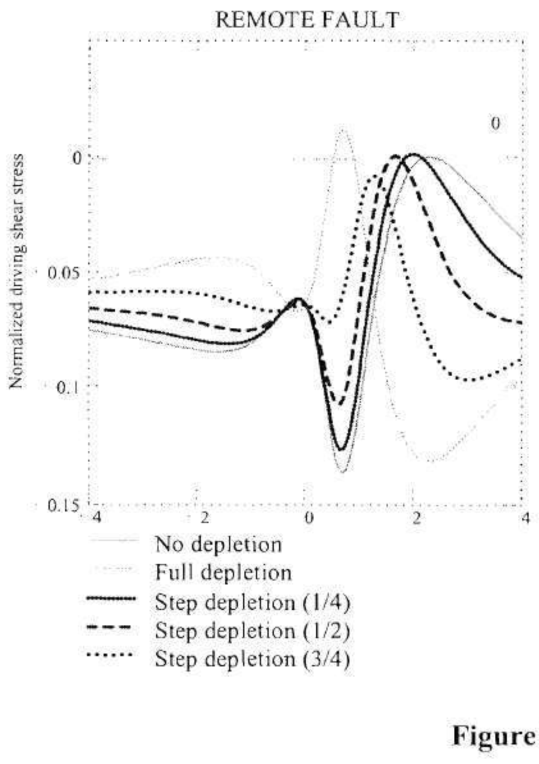
$\theta = 60 \cdot deg$





Cohesion $C_{1,1} = 11.57 \cdot MPa$, **no friction**

No cohesion, friction $\phi_{2,1} = 12.56 \cdot deg$



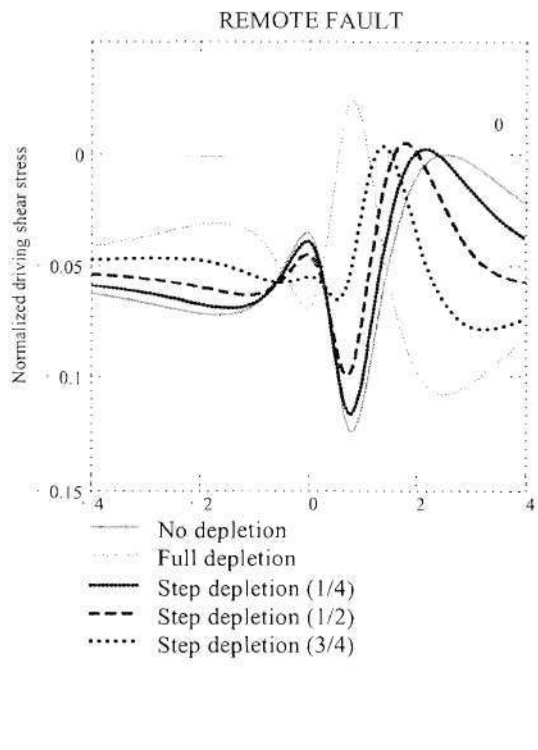


Figure 8.7.

NOTE: The figures above are plotted using the friction and cohesion computed in Sections 7.1 and 7.2 for the fault at the verge of instability.

Reference Parameters from Section 1:

$E = 1 \cdot 10^{10} \cdot \mathrm{Pa}$

$\nu = 0.25$

$\nu_1 = 0.35$

$\frac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\frac{p_0}{\sigma_0} = 1$

$\frac{\sigma_1}{\sigma_0} = -1$

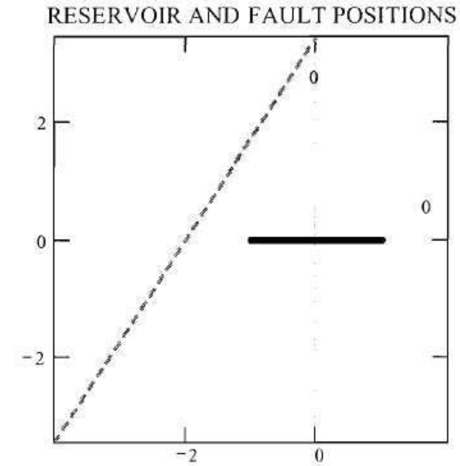
$\lambda = 0.75$

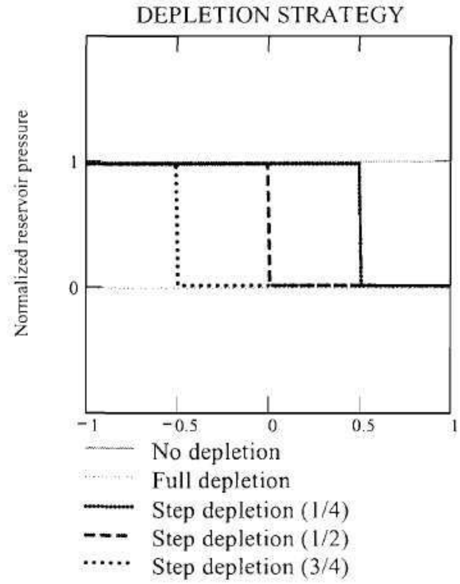
$\frac{p_f}{\sigma_0} = 0.3$

$\frac{2 \cdot a}{h_{max}} = 10$

$\gamma = 0 \cdot \mathrm{deg}$

$\theta = 60 \cdot \mathrm{deg}$





User defined cohesion $C_{31} = 5.8 \cdot \mathrm{MPa}$
and friction $\phi_{3,1} = 6.3 \cdot \mathrm{deg}$

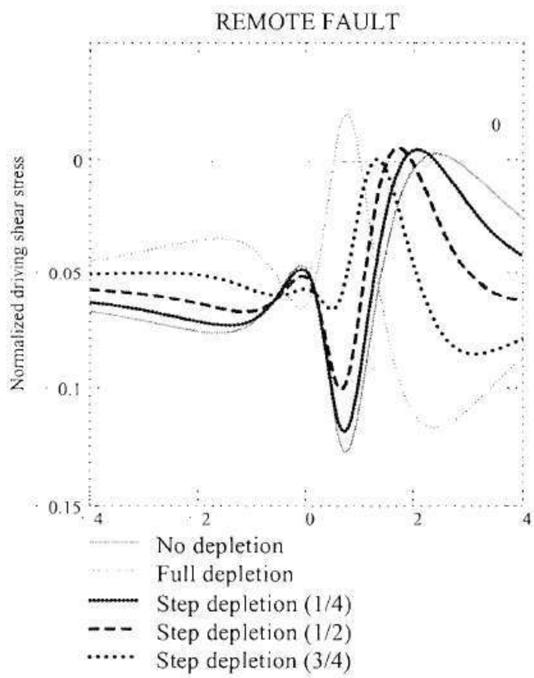


Figure 8.8.

NOTE: The figure above is plotted using the friction and cohesion identified by the user in Section 7.3.

Reference Parameters from Section 1:

$E = 1 \cdot 10^{10} \cdot Pa$

$\nu = 0.25$

$\nu_1 = 0.35$

$\frac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\frac{p_0}{\sigma_0} = 1$

$\frac{\sigma_1}{\sigma_0} = 1$

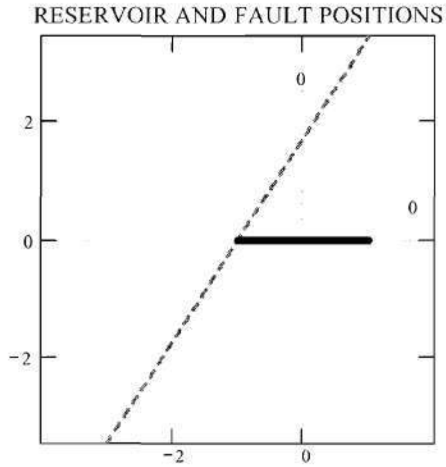
$\lambda = 0.75$

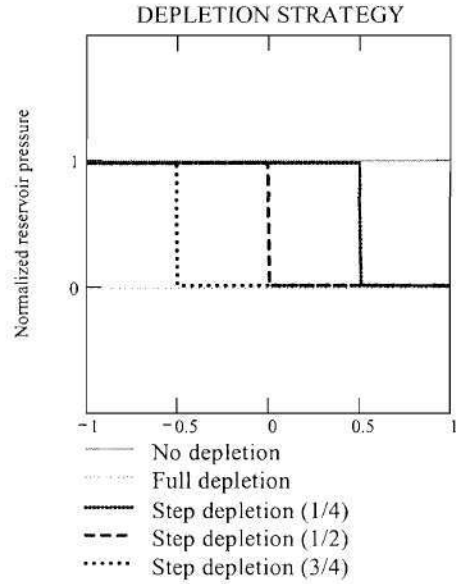
$\frac{p_f}{\sigma_0} = 0.3$

$\frac{2 \cdot a}{h_{max}} = 10$

$\gamma = 0 \cdot deg$

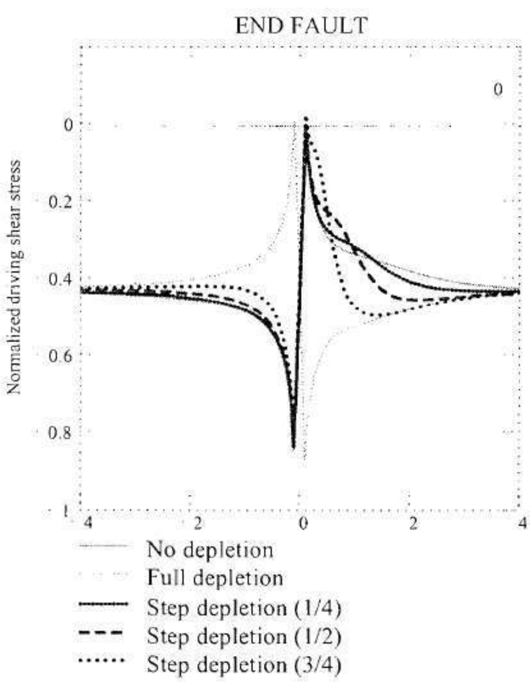
$\theta = 60 \cdot deg$





Cohesion $C_{1,2} = 18.89 \cdot MPa$**, no friction**

No cohesion, friction $\phi_{2,2} = 17.75 \cdot deg$



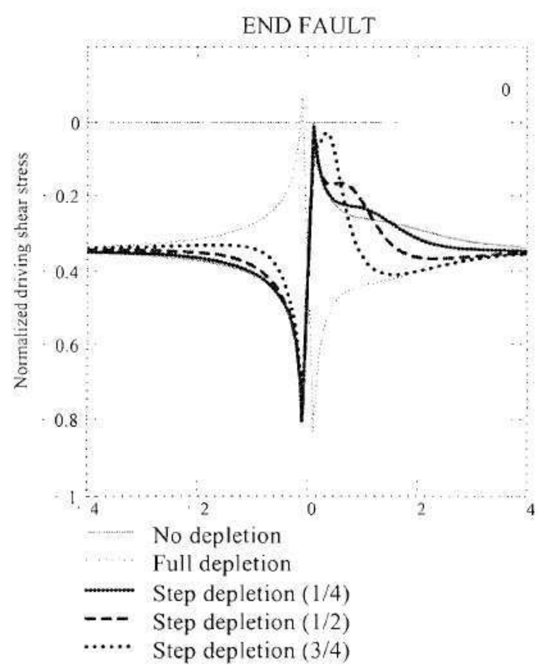
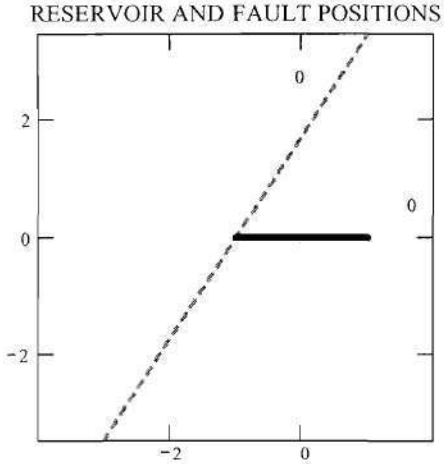
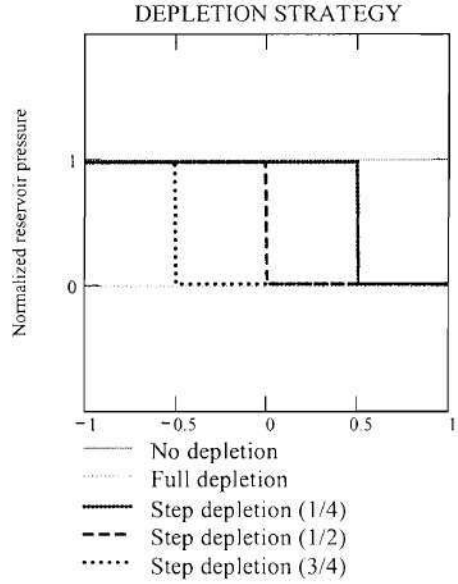


Figure 8.9.

NOTE: The figures above are plotted using the friction and cohesion computed in Sections 7.1 and 7.2 for the fault at the verge of instability.

Reference Parameters from Section 1:





$E = 1 \cdot 10^{10} \cdot Pa$

$\nu = 0.25$

$\nu_1 = 0.35$

$\frac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\frac{p_0}{\sigma_0} = 1$

$\frac{\sigma_1}{\sigma_0} = 1$

$\lambda = 0.75$

$\frac{p_f}{\sigma_0} = 0.3$

$\frac{2 \cdot a}{h_{max}} = 10$

$\gamma = 0 \cdot deg$

$\theta = 60 \cdot deg$

User defined cohesion $C_{3,1} = 5.8 \cdot MPa$
and friction $\phi_{3,1} = 6.3 \cdot deg$

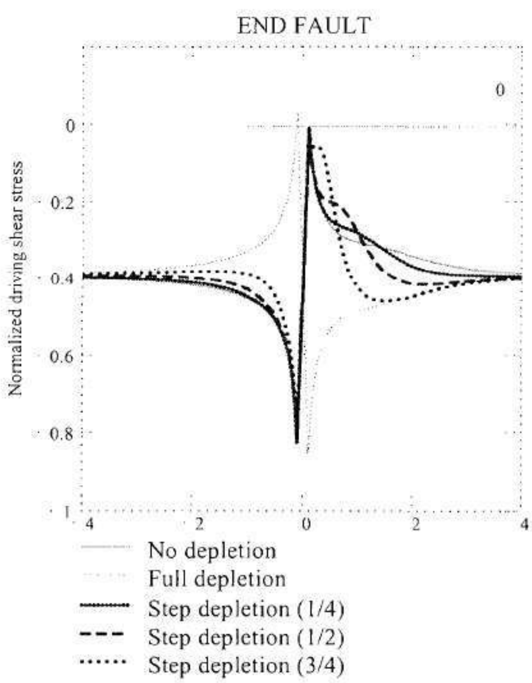


Figure 8.10.

NOTE: The figure above is plotted using the friction and cohesion identified by the user in Section 7.3.

Reference Parameters from Section 1:

$E = 1 \cdot 10^{10} \cdot Pa$

$\nu = 0.25$

$\nu_1 = 0.35$

$\frac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\frac{p_0}{\sigma_0} = 1$

$\frac{\sigma_1}{\sigma_0} = -1$

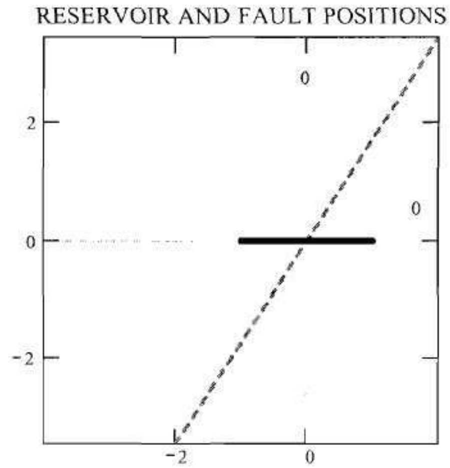
$\lambda = 0.75$

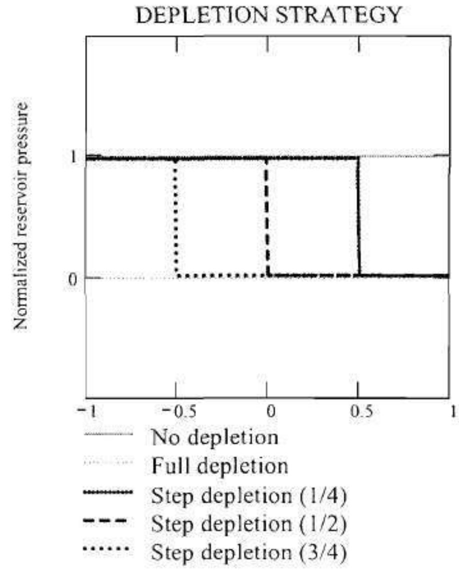
$\frac{p_f}{\sigma_0} = 0.3$

$\frac{2 \cdot a}{h_{max}} = 10$

$\gamma = 0 \cdot deg$

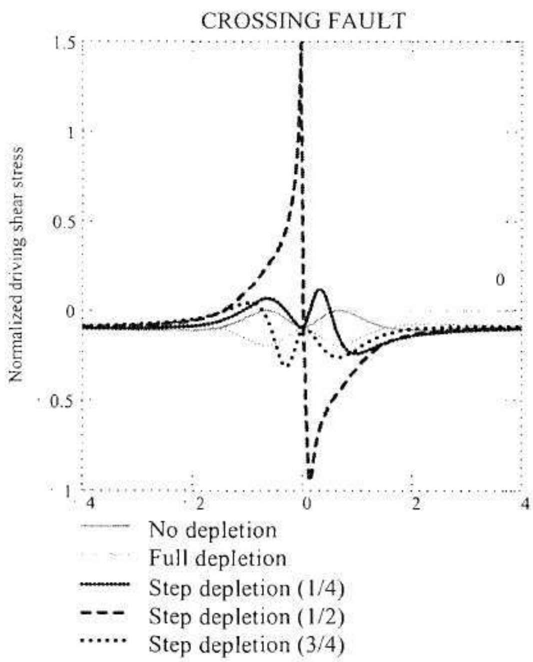
$\theta = 60 \cdot deg$





Cohesion $C_{1,3} = 11.98 \cdot MPa$, **no friction**

No cohesion, friction $\phi_{2,3} = 12.93 \cdot deg$



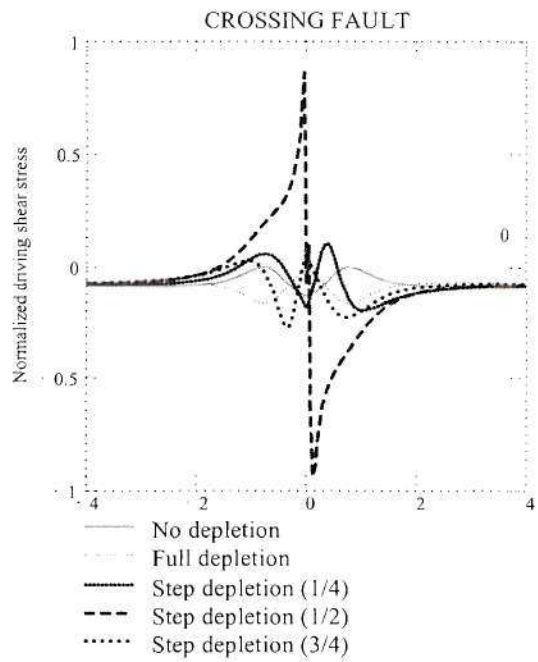


Figure 8.11.

NOTE: The figures above are plotted using the friction and cohesion computed in Sections 7.1 and 7.2 for the fault at the verge of instability.

Reference Parameters from Section 1:

$E = 1 \cdot 10^{10} \cdot Pa$

$\nu = 0.25$

$\nu_1 = 0.35$

$\frac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\frac{p_0}{\sigma_0} = 1$

$\frac{\sigma_1}{\sigma_0} = 1$

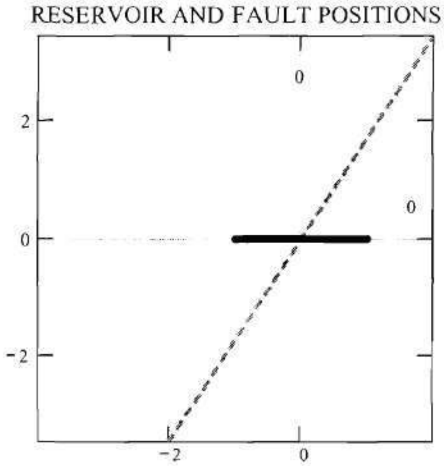
$\lambda = 0.75$

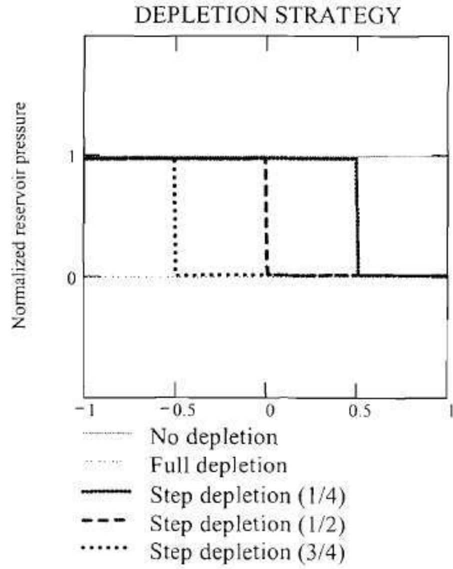
$\frac{p_f}{\sigma_0} = 0.3$

$\frac{2 \cdot a}{h_{max}} = 10$

$\gamma = 0 \cdot deg$

$\theta = 60 \cdot deg$





User defined cohesion $C_{3,3} = 5.9 \cdot MPa$
and friction $\phi_{3,3} = 6.5 \cdot deg$

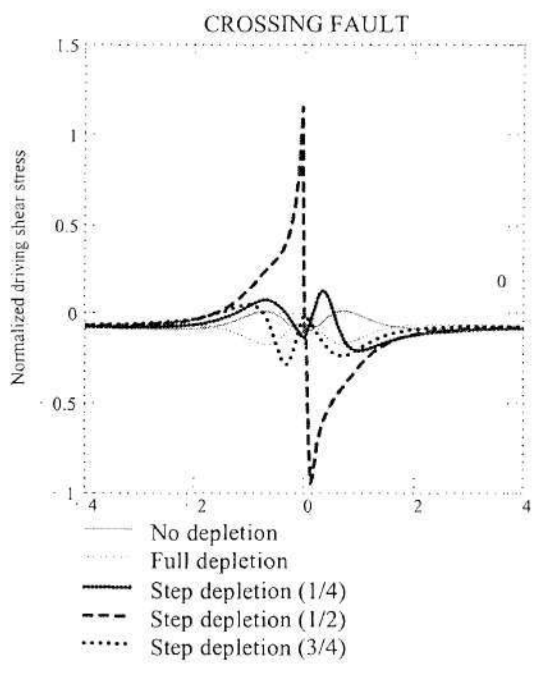


Figure 8.12.

NOTE: The figure above is plotted using the friction and cohesion identified by the user in Section 7.3.

9. Fault Slip and Seismicity

NOTE: For the user convenience, in Section 9.1 the user is given a choice to redefine the strength parameters (i.e., friction coefficient and cohesion). This is why this subsection mainly repeats Section 8.2. This may be a useful option allowing the user to independently test other parameters before coming to the most time consuming calculations programmed in Section 9.2.

9.1. Driving Shear Stresses for Arbitrary Friction and Cohesion Before the Slip (Depletion/Injection)

$f_{\Delta p}(\xi) := 1$

$\Delta p_{max} := 0 \cdot p_0$

$\phi_{arbitrary} := \begin{bmatrix} 12.55 \\ 12.93 \end{bmatrix} \cdot deg$ $\phi_{arbitrary} = \begin{bmatrix} 12.55 \\ 12.93 \end{bmatrix} \circ deg$

$\phi_{arbitrary\_end} := 18.07 \cdot deg$ $\phi_{arbitrary\_end} = 18.07 \circ deg$

$C_{arbitrary} := \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot MPa$ $C_{arbitrary} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \circ MPa$

$C_{arbitrary\_end} := 0 \cdot MPa$ $C_{arbitrary\_end} = 0 \circ MPa$

NOTE: If the user wishes to change the above values for friction angle and cohesion, the new values should be typed in directly instead of those highlighted above.

The following calculations are for evaluating the coefficients α and β:

$j := 1, 2 .. N$

$R_{\alpha 1_j} := f_{\Delta p}(\xi_j)$

$R_{\alpha 2_j} := 1$

$$R_{\alpha}(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := R_{\alpha 1} \cdot \frac{\alpha_p \cdot \Delta p_{max}}{\sigma_0} \ldots$$
$$+ R_{\alpha 2} \cdot \frac{\begin{array}{l}(1 - \varepsilon_1(E, E_1, \nu, \nu_1)) \cdot \sigma_{yy\_remote}(\gamma, \lambda, \sigma_1) \ldots \\ + (-1) \cdot \varepsilon_k(E, E_1, \nu, \nu_1) \cdot \sigma_{xx\_remote}(\gamma, \lambda, \sigma_1) + \alpha_p \cdot p_0\end{array}}{\sigma_0}$$

$$\varepsilon_{\alpha}(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \text{lsolve}(A(E, E_1, \nu, \nu_1, h_{max}, a), R_{\alpha}(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p))$$

$$\alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \varepsilon_{\alpha}(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) \cdot \sigma_0$$

$j := 1, 2 .. \text{last}(\xi_f)$

$\alpha := \alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$

$\beta := \beta'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$

Coulomb shear stress along the fault lines before depletion/injection is defined by:

$$\tau_{Clmb\_arbitrary\_1_{i1,2}} := \tau_{Coulomb}\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_2}, p_f\right)$$

$$\tau_{arbitrary\_1_{i1,2}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_2}, C_{arbitrary_2}, p_f\right)$$

Coulomb shear stress and strength along the fault lines as a result of *full* depletion are: $\Delta p_{max} := -1 \cdot p_0$

$$\alpha := \alpha'\left(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right) \qquad \beta := \beta'\left(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right)$$

$$\tau_{Clmb\_arbitrary\_3_{i1,2}} := \tau_{Coulomb}\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_2}, p_f\right)$$

$$\tau_{arbitrary\_3_{i1,2}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_2}, C_{arbitrary_2}, p_f\right)$$

Reservoir is depleted by a *step-like* distribution of pressure at the right quarter:

$$f_{\Delta p}(\xi) := \begin{cases} 1 & \text{if } \xi \ge 0.5 \\ 0 & \text{otherwise} \end{cases}$$

NOTE: These two expressions should be disabled and replaced by the desired pressure distribution if needed. The second formula is used only for the convenience of plotting.

$$f_{\Delta p\_1}(\xi) := \begin{cases} 0.98 & \text{if } \xi \ge 0.5 \\ 0.02 & \text{otherwise} \end{cases}$$

The following calculations are for evaluating the coefficients α and β:

$$j := 1, 2 .. N$$

$$R_{\alpha 1_j} := f_{\Delta p}\left(\xi_j\right)$$

$$R_{\alpha 2_j} := 1$$

$$R_\alpha\left(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right) := R_{\alpha 1} \cdot \frac{\alpha_p \cdot \Delta p_{max}}{\sigma_0} \ldots \\ + R_{\alpha 2} \cdot \frac{1 - \varepsilon_1\left(E, E_1, \nu, \nu_1\right) \cdot \sigma_{yy\_remote}\left(\gamma, \lambda, \sigma_1\right) \ldots \\ + (-1) \cdot \varepsilon_k\left(E, E_1, \nu, \nu_1\right) \cdot \sigma_{xx\_remote}\left(\gamma, \lambda, \sigma_1\right) + \alpha_p \cdot p_0}{\sigma_0}$$

$$\varepsilon_\alpha\left(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right) := \text{lsolve}\left(A\left(E, E_1, \nu, \nu_1, h_{max}, a\right), R_\alpha\left(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right)\right)$$

$$\alpha'\left(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right) := \varepsilon_\alpha\left(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right) \cdot \sigma_0$$

$j := 1, 2 .. \text{last}(\xi_f)$

$\alpha := \alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$

$\beta := \beta'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$

Coulomb shear stress and strength along the crossing fault line as a result of step-like distribution of pressure depletion at the right quarter of the reservoir are:

$\tau_{Clmb\_arbitrary\_rev\_step\_1_{i1,2}} := \tau_{Coulomb}(\zeta_{i1,2}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_2}, p_f)$

$\tau_{arbitrary\_rev\_step\_1_{i1,2}} := \tau_f(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_2}, C_{arbitrary_2}, p_f)$

Reservoir is depleted by a step-like distribution of pressure at the center from the right:

$$f_{\Delta p}(\xi) := \begin{cases} 1 & \text{if } \xi \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

NOTE: These two expressions should be disabled and replaced by the desired pressure distribution if needed. The second formula is used only for the convenience of plotting.

$$f_{\Delta p\_2}(\xi) := \begin{cases} 0.98 & \text{if } \xi \geq 0 \\ 0.02 & \text{otherwise} \end{cases}$$

The following calculations are for evaluating the coefficients α and β:

$j := 1, 2 .. N$

$R_{\alpha 1_j} := f_{\Delta p}(\xi_j)$

$R_{\alpha 2_j} := 1$

$$R_\alpha(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := R_{\alpha 1} \cdot \frac{\alpha_p \cdot \Delta p_{max}}{\sigma_0} \ldots \\ + R_{\alpha 2} \cdot \frac{(1 - \varepsilon_1(E, E_1, \nu, \nu_1)) \cdot \sigma_{yy\_remote}(\gamma, \lambda, \sigma_1) \ldots + (-1) \cdot \varepsilon_k(E, E_1, \nu, \nu_1) \cdot \sigma_{xx\_remote}(\gamma, \lambda, \sigma_1) + \alpha_p \cdot p_0}{\sigma_0}$$

$\varepsilon_\alpha(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \text{lsolve}(A(E, E_1, \nu, \nu_1, h_{max}, a), R_\alpha(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p))$

$\alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \varepsilon_\alpha(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) \cdot \sigma_0$

$j := 1, 2 .. \text{last}(\xi_f)$

$\alpha := \alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$ $\quad \alpha_1 := \alpha$

$\beta := \beta'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$ $\quad \beta_1 := \beta$

NOTE: The coefficients α_1 and β_1 will be used in the calculation of stress intensity factors and displacement distributions in Section 9.2.

Coulomb shear stress and strength along the crossing fault line as a result of step-like distribution of pressure depletion at the right half of the reservoir are:

$\tau_{Clmb\_arbitrary\_rev\_step\_2_{i1,2}} := \tau_{Coulomb}\left(\zeta_{i1,2}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_2}, p_f\right)$

$\tau_{arbitrary\_rev\_step\_2_{i1,2}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_2}, C_{arbitrary_2}, p_f\right)$

Reservoir is depleted by a step-like distribution of pressure at the right three quarters:

$f_{\Delta p}(\xi) := \begin{vmatrix} 1 & \text{if } \xi \geq -0.5 \\ 0 & \text{otherwise} \end{vmatrix}$

NOTE: These two expressions should be disabled and replaced by the desired pressure distribution if needed. The second formula is used only for the convenience of plotting.

$f_{\Delta p\_3}(\xi) := \begin{vmatrix} 0.98 & \text{if } \xi \geq -0.5 \\ 0.02 & \text{otherwise} \end{vmatrix}$

The following calculations are for evaluating the coefficients α and β:

$j := 1, 2 .. N$

$R_{\alpha 1_j} := f_{\Delta p}\left(\xi_j\right)$

$R_{\alpha 2_j} := 1$

$$R_\alpha\left(E, E_1, v, v_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right) := R_{\alpha 1} \cdot \frac{\alpha_p \cdot \Delta p_{max}}{\sigma_0} \dots \\ + R_{\alpha 2} \cdot \frac{\left(1 - \varepsilon_1\left(E, E_1, v, v_1\right)\right) \cdot \sigma_{yy\_remote}\left(\gamma, \lambda, \sigma_1\right) \dots + (-1) \cdot \varepsilon_k\left(E, E_1, v, v_1\right) \cdot \sigma_{xx\_remote}\left(\gamma, \lambda, \sigma_1\right) + \alpha_p \cdot p_0}{\sigma_0}$$

$\varepsilon_\alpha\left(E, E_1, v, v_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right) := \text{lsolve}\left(A\left(E, E_1, v, v_1, h_{max}, a\right), R_\alpha\left(E, E_1, v, v_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right)\right)$

$\alpha'\left(E, E_1, v, v_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right) := \varepsilon_\alpha\left(E, E_1, v, v_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right) \cdot \sigma_0$

$j := 1, 2 .. \text{last}\left(\xi_f\right)$

$\alpha := \alpha'\left(E, E_1, v, v_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right)$

$\beta := \beta'\left(E, E_1, v, v_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p\right)$

Coulomb shear stress and strength along the fault lines as a result of step-like distribution of pressure depletion at the right three quarters of the reservoir are:

$\tau_{Clmb\_arbitrary\_rev\_step\_3_{i1,2}} := \tau_{Coulomb}\left(\zeta_{i1,2}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_2}, p_f\right)$

$\tau_{arbitrary\_rev\_step\_3_{i1,2}} := \tau_f\left(\zeta_{i1,j}, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta, \phi_{arbitrary_2}, C_{arbitrary_2}, p_f\right)$

Reference Parameters from Section 1:

$E = 1 \cdot 10^{10} \cdot Pa$

$\nu = 0.25$

$\nu_1 = 0.35$

$\frac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\frac{p_0}{\sigma_0} = 1$

$\frac{\sigma_1}{\sigma_0} = 1$

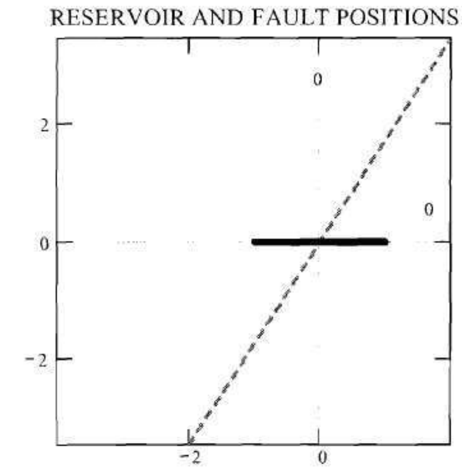
$\lambda = 0.75$

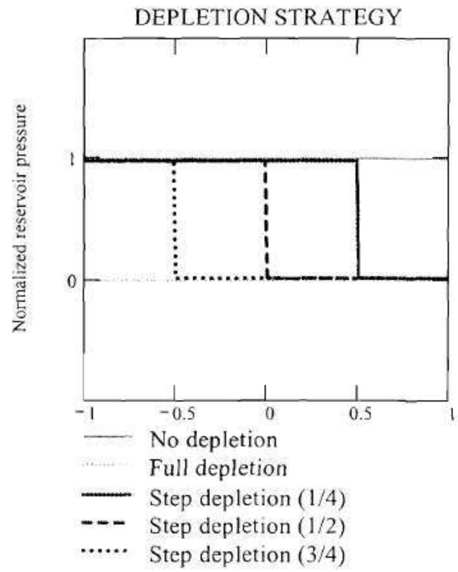
$\frac{p_f}{\sigma_0} = 0.3$

$\frac{2 \cdot a}{h_{max}} = 10$

$\gamma = 0 \cdot deg$

$\theta = 60 \cdot deg$





User defined cohesion $c_{arbitrary_2} = 0 \cdot MPa$

and friction $\phi_{arbitrary_2} = 12.93 \cdot deg$

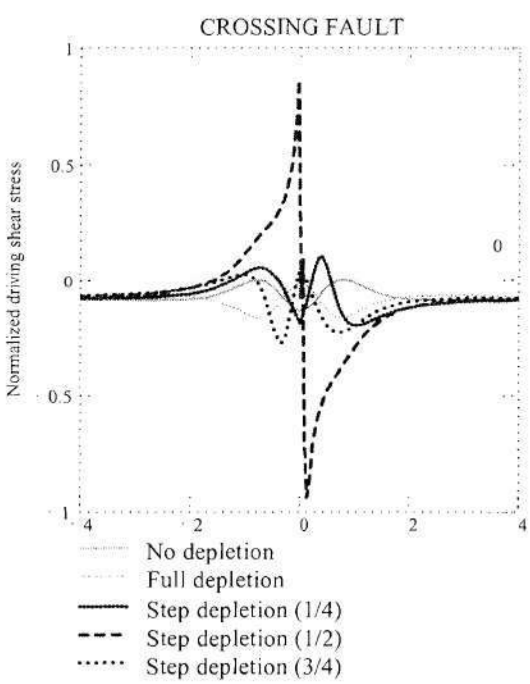


Figure 9.1.

NOTE: The figure above is plotted using the friction and cohesion identified by the user in this section.

9.2. Fault Slip Magnitude

The fault slips over the region where the driving shear stress, $\Delta\tau(x)$, is greater than zero. As a first approximation, the amount of slip over the slipped regions can be calculated using the following formula (e.g., Tada, 1985):

$$2\cdot u(x)=\frac{1-\nu}{\mu}\cdot\frac{2}{\pi}\cdot\int_{-c}^{c}\Delta\tau(\xi)\cdot a\cdot\ln\left(\frac{\sqrt{1+\xi}\cdot\sqrt{1-x}+\sqrt{1-\xi}\cdot\sqrt{1+x}}{\sqrt{1+\xi}\cdot\sqrt{1-x}-\sqrt{1-\xi}\cdot\sqrt{1+x}}\right)d\xi \qquad (9.1)$$

where $\Delta\tau(x)$ is the driving shear stress and c is the half-length of the region where the fault slips.

The stress intensity factor at the tips of the slipped portions of the fault is calculated using the following formula (e.g., Tada, 1985):

$$K_{II\_L}=\frac{1}{\sqrt{\pi\cdot c\cdot a}}\cdot\int_{-c}^{c}\tau(\xi)\cdot a\cdot\sqrt{\frac{c-\xi}{c+\xi}}\,d\xi \qquad (9.2)$$

$$K_{II\_R}=\frac{1}{\sqrt{\pi\cdot c\cdot a}}\cdot\int_{-c}^{c}\tau(\xi)\cdot a\cdot\sqrt{\frac{c+\xi}{c-\xi}}\,d\xi \qquad (9.3)$$

$a_1 := -1.6$ Initial coordinate of the left tip of the slipping portion of the crossing fault (defined by observation from driving shear stress distribution plot in Section 9.1).

$a_2 := 0$ Initial coordinate of the right tip of the slipping portion of the crossing fault (defined by observation from driving shear stress distribution plot in Section 9.1).

NOTE: Mathcad 8 Pro utility [zoom] allows to define the coordinate of a point on a plot very accurately.

$\Delta a_1 := 0$ Increment in the slipping portion at the left tip depending upon stress intensity factor at left tip.

$\Delta a_2 := 0$ Increment in slipping portion at the right tip depending upon stress intensity factor at right tip.

NOTE: Initially, Δa_1 and Δa_2 are taken as zero. Eventually though these are chosen such that the stress intensity factor, K_{II} becomes zero, i.e., to obtain an upper estimate of the fault slip.

$a_1 := a_1 - \Delta a_1$ Coordinate of the left tip of the slipped portion of the fault, $a_1 = -1.6$ (9.3a)

$a_2 := a_2 + \Delta a_2$ Coordinate of the right tip of the slipped portion of the fault, $a_2 = 0$ (9.3b)

$c := \dfrac{a_1 - a_2}{2}$ Half-size of the slipped portion of the fault, $c = 0.8$ (9.3c)

$\xi'_f := \dfrac{a_1 + a_2}{2}$ Coordinate of the center of the slipped portion of the fault, $\xi'_f = -0.8$ (9.3d)

Define the driving shear stress along the fault taking into account the sign of shear stress:

$$SIGN_\tau(\xi) := \mathrm{sign}\left[\tau_{x'y'}\left[\xi_{f_2} + (\xi + \xi'_f)\cdot\cos(\theta) + i\cdot(\xi + \xi'_f)\cdot\sin(\theta), \theta, \gamma, \lambda, \sigma_1, \alpha_1, \beta_1\right]\right] \quad (9.3e)$$

$$\tau(\xi) := \left[\tau_{Coulomb}\left[\xi_{f_2} + (\xi + \xi'_f)\cdot\cos(\theta) + i\cdot(\xi + \xi'_f)\cdot\sin(\theta), \theta, \gamma, \lambda, \sigma_1, \alpha_1, \beta_1, \phi_{arbitrary_2}, p_f\right] - C_{arbitrary_2}\right]\cdot SIGN_\tau(\xi)$$

(9.3f)

$K_{II\_L} := \dfrac{1}{\sqrt{\pi\cdot c\cdot a}}\cdot\displaystyle\int_{-c}^{c}\tau(\xi)\cdot a\cdot\sqrt{\dfrac{c-\xi}{c+\xi}}\,d\xi$ Stress intensity factor at the left tip of the fault (9.3g)

$K_{II\_L} = -22.4867\cdot MPa\cdot\sqrt{m}$

$K_{II\_R} := \dfrac{1}{\sqrt{\pi\cdot c\cdot a}}\cdot\displaystyle\int_{-c}^{c}\tau(\xi)\cdot a\cdot\sqrt{\dfrac{c+\xi}{c-\xi}}\,d\xi$ Stress intensity factor at the right tip of the fault (9.3h)

$K_{II\_R} = -101.5646\cdot MPa\cdot\sqrt{m}$

$u(x) := \dfrac{2\cdot(1-\nu^2)}{E\cdot\pi}\cdot\displaystyle\int_{-c}^{c}\tau(\xi)\cdot a\cdot\ln\left(\left|\dfrac{\sqrt{c+\xi}\cdot\sqrt{c-x}+\sqrt{c-\xi}\cdot\sqrt{c+x}}{\sqrt{c+\xi}\cdot\sqrt{c-x}-\sqrt{c-\xi}\cdot\sqrt{c+x}}\right|\right)d\xi$ Distribution of fault slip magnitudes along the fault (9.3i)

$width := 2\cdot c\cdot a$ Width (lateral dimension) of the slipped portion of the fault. It is assumed that it is same as the total size of the slipped portion of the fault.

$MM := 20$ Number of data points along the slipped portion of the fault (to be used for plotting)

$ii := 1, 2 .. MM + 1$

$xx_{ii} := -c + \dfrac{2\cdot c\cdot(ii-1)}{MM}$

Calculating slip magnitude at each data point

$U_{ii} := u(xx_{ii})$

Calculating driving shear stress at each data point taking into account the sign of shear stress

$T_{ii} := \tau\ xx_{ii}$

$U_1 := \max(U)$

$U_2 := \min(U)$

Defining maximum slip magnitude along the slipped portion of the fault

$$U_{max} := \begin{cases} U_1 & \text{if } |U_1| \geq |U_2| \\ |U_2| & \text{otherwise} \end{cases} \tag{9.3j}$$

$2 \cdot U_{max} = 13.532 \cdot cm$

$T_1 := \max(T)$

$T_2 := \min(T)$

Defining maximum driving shear stress magnitude along the slipped portion of the fault

$$T_{max} := \begin{cases} T_1 & \text{if } |T_1| \geq |T_2| \\ |T_2| & \text{otherwise} \end{cases} \tag{9.3k}$$

Writing the slip magnitudes into a data fi $\mathrm{WRITEPRN}(\text{"UDR\_U1b.txt"}) := \dfrac{U}{a}$

Writing the driving shear magnitudes into a data f $\mathrm{WRITEPRN}(\text{"UDR\_T1b.txt"}) := \dfrac{T}{\sigma_0}$

Writing the data points along the slipped portion of the fault into a dat $\mathrm{WRITEPRN}(\text{"UDR\_X1b.txt"}) := xx + \xi'_f$

NOTE: The above three files save the data of displacement distribution into a text file so that one need not calculate the displacements again. This is done since the time required to calculate slip displacement distribution may be quite long. To retrieve the data , use the command, READPRN("file") .

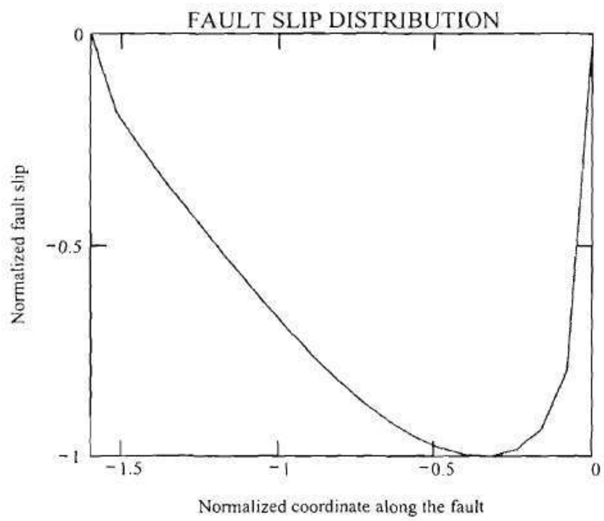


Figure 9.1.

Seismic moment, M_0, is given by:

$M_0 = \mu \cdot A \cdot D$ μ is the shear modulus of the surrounding rock, A is the area of slipped portion of the fault and D is the average slip. (9.4)

$$M_0 := \frac{E}{2\cdot(1+\nu)}\cdot\frac{c\cdot a}{MM}\cdot width\cdot\left(\left|U_1 + U_{MM+1}\right| + 2\cdot\sum_{ii=2}^{MM}\left|U_{ii}\right|\right) \quad (9.4a)$$

$M_0 = 1.0669\cdot10^{13}$ newton·m

Seismic moment magnitude is given by $M_w = 0.69\cdot\log(M_0) - 6.4$, where M_0 is the seismic moment in dyne-cm (Hanks and Kanamori, 1979).

$$M_w := \frac{2}{3}\cdot\log\left(\frac{M_0}{dyne\cdot cm}\right) - 10.7 \quad (9.5)$$

NOTE: This empirical formula relating seismic magnitude and seismic moment is of general nature and is not site specific. This is the user responsibility to find more relevant relationship, which is important should the magnitude of earthquake be of concern.

$M_w = 2.65$

$\Delta a_1 := \frac{5}{6}\cdot 2\cdot c$ Increment in slipping portion at the left tip depending upon stress intensity factor at left tip. (9.5a)

$\Delta a_2 := \frac{2.5}{6}\cdot 1\cdot c$ Increment in slipping portion at the right tip depending upon stress intensity factor at right tip. (9.5b)

NOTE: Initially, Δa_1 and Δa_2 were taken as zero. These are now chosen such that the stress intensity factor, K_{II}, becomes zero (small), i.e., to obtain an upper estimate of the fault slip. The user has to manually change Δa_1 and Δa_2 values until the desired values of K_{II}'s are reached at both the ends (tips) of the slipped zone.

$a_1 := a_1 - \Delta a_1$ Coordinate of the left tip of the slipped portion of the fault, $a_1 = -2.9333$ (9.5c)

$a_2 := a_2 + \Delta a_2$ Coordinate of the right tip of the slipped portion of the fault, $a_2 = 0.3333$ (9.5d)

$c := \left|\dfrac{a_1 - a_2}{2}\right|$ Half-length of the slipped portion of the fault, $c = 1.6333$ (9.5e)

$\xi'_f := \dfrac{a_1 + a_2}{2}$ Coordinate of the center of the slipped portion of the fault, $\xi'_f = -1.3$ (9.5f)

Define driving shear stress along the fault taking into account the sign of shear stress:

$$\mathrm{SIGN}_\tau(\xi) := \mathrm{sign}\left[\tau_{x'y'}\left[\xi_{f_2} + (\xi + \xi'_f)\cdot\cos(\theta) + i\cdot(\xi + \xi'_f)\cdot\sin(\theta), \theta, \gamma, \lambda, \sigma_1, \alpha_1, \beta_1\right]\right] \quad (9.5g)$$

$$\tau(\xi) := \left[\tau_{\mathrm{Coulomb}}\left[\xi_{f_2} + (\xi + \xi'_f)\cdot\cos(\theta) + i\cdot(\xi + \xi'_f)\cdot\sin(\theta), \theta, \gamma, \lambda, \sigma_1, \alpha_1, \beta_1, \phi_{\mathrm{arbitrary}_2}, p_f\right] - C_{\mathrm{arbitrary}_2}\right]\cdot \mathrm{SIGN}_\tau(\xi) \quad (9.5h)$$

$$K_{II\_L} := \frac{1}{\sqrt{\pi\cdot c\cdot a}}\cdot\int_{-c}^{c}\tau(\xi)\cdot a\cdot\sqrt{\frac{c-\xi}{c+\xi}}\,d\xi$$ Stress intensity factor at the left tip of the fault (9.5i)

$K_{II\_L} = 0.147\,\mathrm{MPa}\cdot\sqrt{m}$

$$K_{II\_R} := \frac{1}{\sqrt{\pi\cdot c\cdot a}}\cdot\int_{-c}^{c}\tau(\xi)\cdot a\cdot\sqrt{\frac{c+\xi}{c-\xi}}\,d\xi$$ Stress intensity factor at the right tip of the fault (9.5j)

$K_{II\_R} = 1.9376\,\mathrm{MPa}\cdot\sqrt{m}$

$$u(x) := \frac{2\cdot(1-\nu^2)}{E\cdot\pi}\cdot\int_{-c}^{c}\tau(\xi)\cdot a\cdot\ln\left(\left|\frac{\sqrt{c+\xi}\cdot\sqrt{c-x}+\sqrt{c-\xi}\cdot\sqrt{c+x}}{\sqrt{c+\xi}\cdot\sqrt{c-x}-\sqrt{c-\xi}\cdot\sqrt{c+x}}\right|\right)d\xi$$ Distribution of fault slip magnitudes along the fault (9.5k)

$\mathrm{width} := 2\cdot c\cdot a$ Width (lateral dimension) of the slipped portion of the fault. It is assumed that it is same as the slipped portion of the fault.

$MM := 20$ Number of data points along the slipped portion of the fault

$ii := 1, 2 .. MM + 1$

$xx_{ii} := -c + \frac{2 \cdot c \cdot (ii - 1)}{MM}$

$U_{ii} := u(xx_{ii})$ Calculating slip magnitude at each data point

$T_{ii} := \tau(xx_{ii})$ Calculating driving shear stress at each data point taking into account the sign of shear stress

$U_1 := \max(U)$

$U_2 := \min(U)$

Defining maximum slip magnitude along the slipped portion of the fault

$$U_{max} := \begin{vmatrix} U_1 & \text{if } |U_1| \geq |U_2| \\ U_2 & \text{otherwise} \end{vmatrix} \tag{9.5l}$$

$2 \cdot U_{max} = 19.047 \cdot cm$

$T_1 := \max(T)$

$T_2 := \min(T)$

Defining maximum driving shear stress magnitude along the slipped portion of the fault

$$T_{max} := \begin{vmatrix} T_1 & \text{if } |T_1| \geq |T_2| \\ T_2 & \text{otherwise} \end{vmatrix} \tag{9.5m}$$

Writing the slip magnitudes into a data file: $\text{WRITEPRN("UDR\_U1d.txt")} := \frac{U}{a}$

Writing the driving shear magnitudes into a data fil $\text{WRITEPRN("UDR\_T1d.txt")} := \frac{T}{\sigma_0}$

Writing the data points along the slipped portion of the fault into a data f $\text{WRITEPRN("UDR\_X1d.txt")} := xx + \xi'_f$

NOTE: The above three files save the data of displacement distribution into a text file so that one need not calculate the displacements again. This is done since the time required to calculate slip displacement distribution may be quite long. To retrieve the data , use the command, READPRN("file")

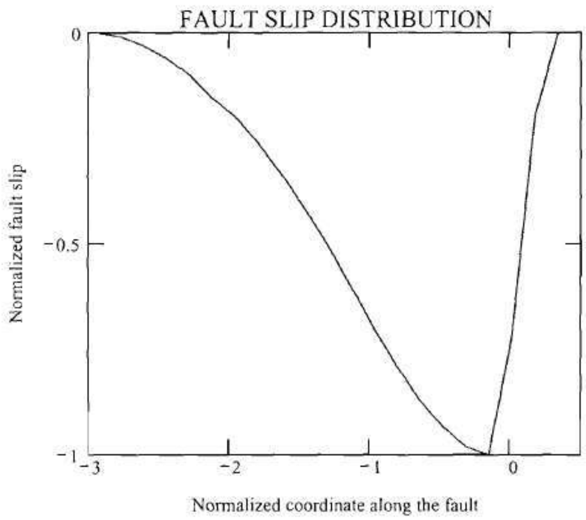


Figure 9.2.

Seismic moment, M_0 is given by:

$M_0 = \mu \cdot A \cdot D$ μ is the shear modulus of the surrounding rock, A is the area of slipped portion of the fault and D is the average slip. (9.5n)

$$M_0 := \frac{E}{2\cdot(1+v)}\cdot\frac{c\cdot a}{MM}\cdot width\cdot\left(\left|U_1 + U_{MM+1}\right| + 2\cdot\sum_{ii=2}^{MM}\left|U_{ii}\right|\right) \qquad (9.5o)$$

$M_0 = 4.0637\cdot 10^{13}\ newton\cdot m$

Seismic moment magnitude is given by $M_w = 0.69\cdot\log(M_0) - 6.4$, where M_0 is the seismic moment in dyne-cm (Hanks and Kanamori, 1979).

$$M_w := \frac{2}{3}\cdot\log\left(\frac{M_0}{dyne\cdot cm}\right) - 10.7 \qquad (9.5p)$$

NOTE: This empirical formula relating seismic magnitude and seismic moment is of general nature and is not site specific. This is the user responsibility to find more relevant relationship, which is important should the magnitude of earthquake be of concern.

$M_w = 3.04$

10. Stress Distribution

Same parameters as defined in Section 1 are used in this section. They are listed below for reference:

$E = 1 \cdot 10^{10} \cdot Pa$ $\qquad \sigma_1 = -100 \cdot MPa$

$E_1 = 0.3333 \cdot E$ $\qquad \lambda = 0.75$

$\nu = 0.25$ $\qquad \sigma_3(\lambda, \sigma_1) = -75 \cdot MPa$

$\nu_1 = 0.35$ $\qquad \alpha_p = 1$

$a = 150\ m$ $\qquad p_0 = 100 \cdot MPa$

$h_{max} = 30\ m$ $\qquad p_f = 30 \cdot MPa$

$\gamma = 0 \cdot deg$

$\theta = 60 \cdot deg$

The depletion pressure drop, $\Delta p(x)$, is set to zero to compute the in-situ principal stresses outside and inside the reservoir before depletion:

$\Delta p_{max} := 0 \cdot p_0$

NOTE: If you wish to compute the stress distribution as a result of depletion, replace $\Delta p(x)$. For example, for full depletion, $\Delta p_{max} = -p_0$.

The following calculations are for evaluating the coefficients α and β:

$j := 1, 2 .. N$ $\qquad U(\zeta, n) := \dfrac{\sin((n+1) \cdot acos(\zeta))}{\sin(acos(\zeta))}$

$$A_{1_{j,n}} := \frac{\sqrt{1 - (\xi_j)^2} \cdot U(\xi_j, n-1)}{f_h(\xi_j) \cdot n}$$

$$I_{1_{j,n}} := I(\xi_j, n)$$

$$G_{1_{j,n}} := G(\xi_j, n)$$

$$A(E, E_1, \nu, \nu_1, h_{max}, a) := \frac{2 \cdot \varepsilon_a(E, E_1, \nu, \nu_1)}{\delta_{max}(h_{max}, a)} \cdot A_1 - I_1 + \varepsilon_c(E, E_1, \nu, \nu_1) \cdot G_1$$

$$R_{\alpha 1_j} := f_{\Delta p}(\xi_j)$$

$$R_{\alpha 2_j} := 1$$

$$R_\alpha(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := R_{\alpha 1} \cdot \frac{\alpha_p \cdot \Delta p_{max}}{\sigma_0} \dots + R_{\alpha 2} \cdot \frac{(1 - \varepsilon_l(E, E_1, \nu, \nu_1)) \cdot \sigma_{yy\_remote}(\gamma, \lambda, \sigma_1) \dots + (-1) \cdot \varepsilon_k(E, E_1, \nu, \nu_1) \cdot \sigma_{xx\_remote}(\gamma, \lambda, \sigma_1) + \alpha_p \cdot p_0}{\sigma_0}$$

$$\varepsilon_\alpha(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \mathrm{lsolve}(A(E, E_1, \nu, \nu_1, h_{max}, a), R_\alpha(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p))$$

$$\alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \varepsilon_\alpha(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) \cdot \sigma_0$$

$$B1_{j,n} := \frac{\sqrt{1 - (\xi_j)^2} \cdot U(\xi_j, n-1)}{f_h(\xi_j) \cdot n}$$

$$B(E, E_1, \nu, \nu_1, h_{max}, a) := \frac{\varepsilon_b(E, E_1, \nu, \nu_1)}{\delta_{max}(h_{max}, a)} \cdot B1 - I1 - \varepsilon_d(E, E_1, \nu, \nu_1) \cdot G1$$

$$R_{\beta 1_j} := 1$$

$$R_\beta(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := R_{\beta 1} \cdot \frac{(1 - \varepsilon_m(E, E_1, \nu, \nu_1)) \cdot \tau_{xy\_remote}(\gamma, \lambda, \sigma_1)}{\sigma_0}$$

$$\varepsilon_\beta(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \mathrm{lsolve}(B(E, E_1, \nu, \nu_1, h_{max}, a), R_\beta(E, E_1, \nu, \nu_1, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p))$$

$$\beta'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) := \varepsilon_\beta(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p) \cdot \sigma_0$$

$$\alpha := \alpha'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$$

$$\beta := \beta'(E, E_1, \nu, \nu_1, h_{max}, a, \gamma, \lambda, \sigma_1, \Delta p_{max}, p_0, \alpha_p)$$

10.1. Stress Distribution Outside the Reservoir

Redefine the stresses outside the reservoir for the sake of convenience (to shorten expressions):

$$\sigma_y(\zeta) := \sigma_{y'y'}(\zeta, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta) \tag{10.1}$$

$$\sigma_x(\zeta) := \sigma_{x'x'}(\zeta, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta) \tag{10.2}$$

$$\tau_{xy}(\zeta) := \tau_{x'y'}(\zeta, \theta, \gamma, \lambda, \sigma_1, \alpha, \beta) \tag{10.3}$$

$$\sigma_{max}(\zeta) := \frac{\sigma_y(\zeta) + \sigma_x(\zeta)}{2} - \sqrt{\left(\frac{\sigma_y(\zeta) - \sigma_x(\zeta)}{2}\right)^2 + \tau_{xy}(\zeta)^2} \tag{10.4}$$

$$\sigma_{min}(\zeta) := \frac{\sigma_y(\zeta) + \sigma_x(\zeta)}{2} + \sqrt{\left(\frac{\sigma_y(\zeta) - \sigma_x(\zeta)}{2}\right)^2 + \tau_{xy}(\zeta)^2} \tag{10.5}$$

NOTE: Because of the chosen sign convention (negative compression) and compressive stress regime, major and minor principal stresses, σ_{max} and σ_{min}, correspond to maximum and minimum *absolute values* of the principal stresses respectively

Two grids of data points around the reservoir are generated to plot the contours. One larger grid "*a*" that covers the entire reservoir and some part around it and one smaller grid "*b*" around the chosen point of the reservoir (e.g., center or tip) to closely examine the principal stresses around this point. Grid "b" zooms in on this point.
Choose the number of grid points in each direction:

$M_a := 19$

$i_a := 1, 2 .. M_a + 1$

$j_a := 1, 2 .. M_a + 1$

$M_b := 9$

$i_b := 1, 2 .. M_b + 1$

$j_b := 1, 2 .. M_b + 1$

Choose the maximum and minimum values of dimensionless coordinates in each direction:

$\xi_{a\_min} := -2$ $\quad \xi_{a\_max} := 2$

$\eta_{a\_min} := -2$ $\quad \eta_{a\_max} := 2$

$$\xi_{a_{i_a}} := \xi_{a\_min} + \frac{\xi_{a\_max} - \xi_{a\_min}}{M_a} \cdot (i_a - 1)$$

$$\eta_{a_{j_a}} := \eta_{a\_min} + \frac{\eta_{a\_max} - \eta_{a\_min}}{M_a} \cdot (j_a - 1)$$

$$\zeta_{a_{i_a, j_a}} := \xi_{a_{i_a}} + \eta_{a_{j_a}} \cdot i$$

$$\sigma_{max\_a_{i_a, j_a}} := \sigma_{max}\left(\zeta_{a_{i_a, j_a}}\right)$$

$$\sigma_{min\_a_{i_a, j_a}} := \sigma_{min}\left(\zeta_{a_{i_a, j_a}}\right)$$

$\xi_{b\_min} := -0.5$ $\quad \xi_{b\_max} := 0.5$

$\eta_{b\_min} := -0.5$ $\quad \eta_{b\_max} := 0.5$

$$\xi_{b_{i_b}} := \xi_{b\_min} + \frac{\xi_{b\_max} - \xi_{b\_min}}{M_b} \cdot (i_b - 1)$$

$$\eta_{b_{j_b}} := \eta_{b\_min} + \frac{\eta_{b\_max} - \eta_{b\_min}}{M_b} \cdot (j_b - 1)$$

$$\zeta_{b_{i_b, j_b}} := \xi_{b_{i_b}} + \eta_{b_{j_b}} \cdot i$$

$$\sigma_{max\_b_{i_b, j_b}} := \sigma_{max}\left(\zeta_{b_{i_b, j_b}}\right)$$

$$\sigma_{min\_b_{i_b, j_b}} := \sigma_{min}\left(\zeta_{b_{i_b, j_b}}\right)$$

10.2. Principal Stress and Local Stress Ratio Contours Outside Horizontal Reservoir

The reservoir is always located at $y = 0$ and $|x| < 1$.

Reference Parameters from Section 1:

$E = 1 \cdot 10^{10} \cdot Pa$

$\nu = 0.25$

$\nu_1 = 0.35$

$\dfrac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\dfrac{p_0}{\sigma_0} = 1$

$\dfrac{\sigma_1}{\sigma_0} = -1$

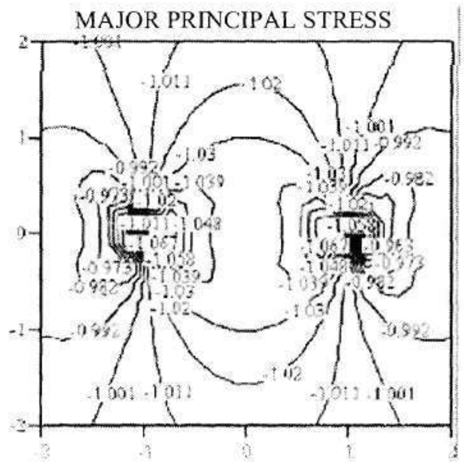
$\lambda = 0.75$

$\dfrac{p_f}{\sigma_0} = 0.3$

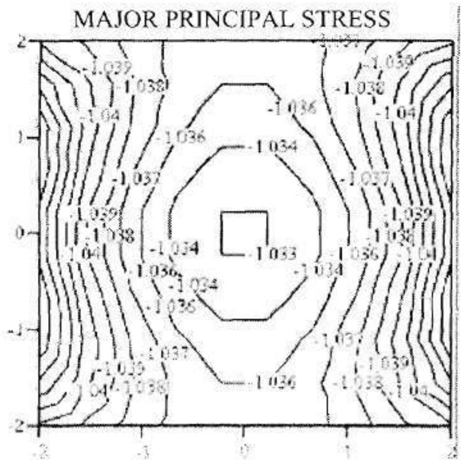
$\dfrac{2 \cdot a}{h_{max}} = 10$

$\gamma = 0 \cdot deg$

$\theta = 60 \cdot deg$



$\dfrac{\sigma_{max\_a}}{\sigma_0}$



$\dfrac{\sigma_{max\_b}}{\sigma_0}$

Figure 10.1.

NOTE: In the case of initial lithostatic stress state, Mathcad 8 Pro may not plot stress contours because the stresses are constant everywhere. In this case, the user will see the graph boxes empty.

Reference
Parameters
from Section 1:

$E = 1 \cdot 10^{10} \cdot Pa$

$\nu = 0.25$

$\nu_1 = 0.35$

$\dfrac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\dfrac{p_0}{\sigma_0} = 1$

$\dfrac{\sigma_1}{\sigma_0} = -1$

$\lambda = 0.75$

$\dfrac{p_f}{\sigma_0} = 0.3$

$\dfrac{2 \cdot a}{h_{max}} = 10$

$\gamma = 0 \cdot deg$

$\theta = 60 \cdot deg$

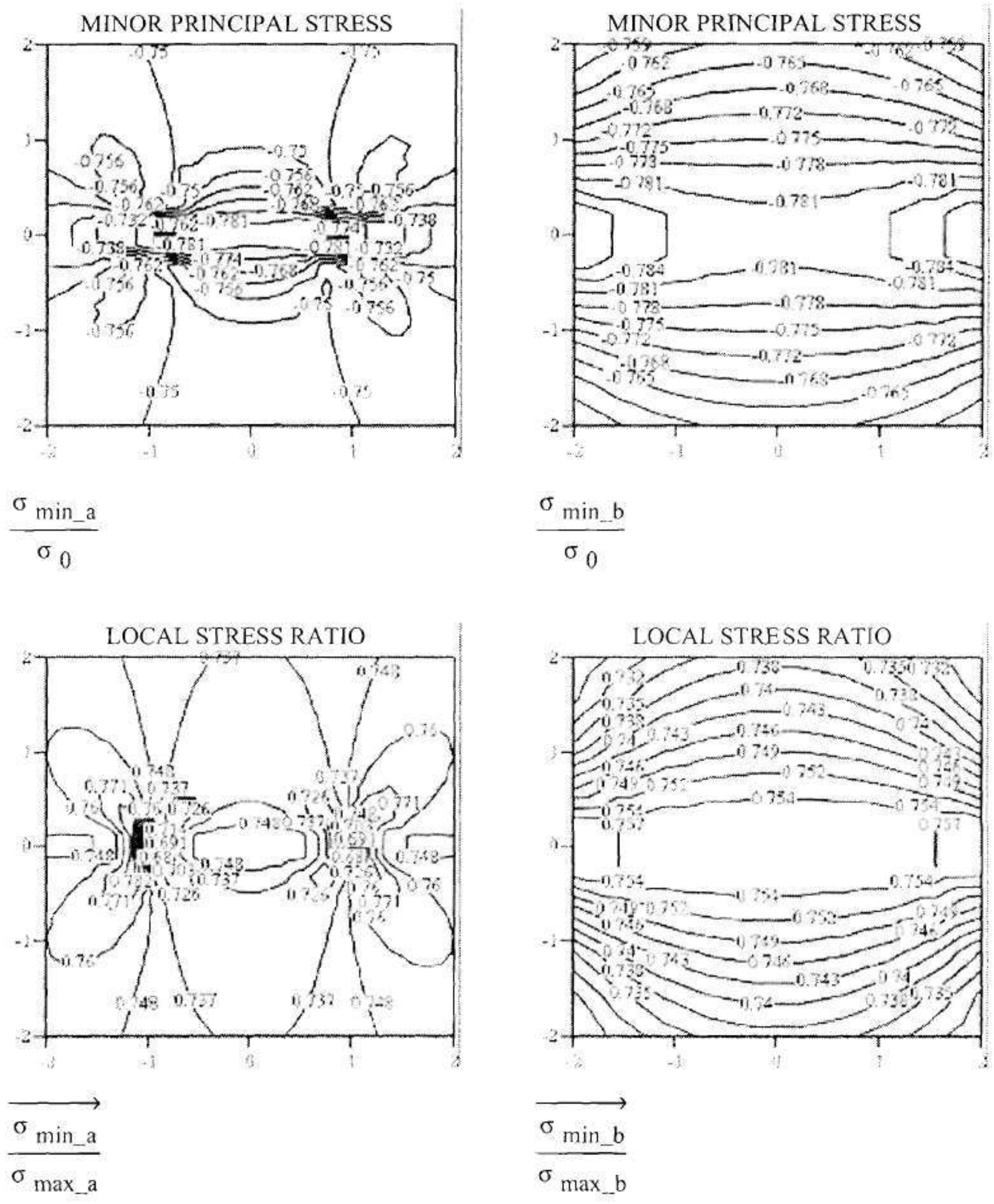


Figure 10.2.

NOTE: In the case of initial lithostatic stress state, Mathcad 8 Pro may not plot stress contours because the stresses are constant everywhere. In this case, the user will see the graph boxes empty.

10.3. Stress Distribution Inside the Reservoir

From Section 4.4:

$$\Delta v(\xi) := \mathrm{Re}\left[\frac{4\cdot a\cdot\left(1-\nu^2\right)}{E}\cdot\sqrt{1-\xi^2}\cdot\sum_{n=1}^{N}\frac{1}{n}\cdot\left(\alpha_n+i\cdot\beta_n\right)\cdot U(\xi,n-1)\right] \tag{10.6}$$

$$\Delta u(\xi) := \mathrm{Im}\left[\frac{4\cdot a\cdot\left(1-\nu^2\right)}{E}\cdot\sqrt{1-\xi^2}\cdot\sum_{n=1}^{N}\frac{1}{n}\cdot\left(\alpha_n+i\cdot\beta_n\right)\cdot U(\xi,n-1)\right] \tag{10.7}$$

$$\sigma_{yy\_res}(\xi) := \frac{E_1\cdot\left(1-\nu_1\right)}{\left(1+\nu_1\right)\cdot\left(1-2\cdot\nu_1\right)}\cdot\frac{\Delta v(\xi)}{h_{max}\cdot f_h(\xi)}+\varepsilon_c\left(E,E_1,\nu,\nu_1\right)\cdot\sum_{n=1}^{N}\alpha_n\cdot G(\xi,n)+\varepsilon_k\left(E,E_1,\nu,\nu_1\right)\cdot\sigma_{xx\_remote}\left(\gamma,\lambda,\sigma_1\right)\ldots$$
$$+\varepsilon_l\left(E,E_1,\nu,\nu_1\right)\cdot\sigma_{yy\_remote}\left(\gamma,\lambda,\sigma_1\right)-\alpha_p\cdot\left(f_{\Delta p}(\xi)\cdot\Delta p_{max}+p_0\right)$$

(10.8)

$$\sigma_{xx\_res}(\xi) := \frac{E_1\cdot\nu_1}{\left(1+\nu_1\right)\cdot\left(1-2\cdot\nu_1\right)}\cdot\frac{\Delta v(\xi)}{h_{max}\cdot f_h(\xi)}+\frac{E_1\cdot\left(1-\nu_1\right)}{\left(1+\nu_1\right)\cdot\left(1-2\cdot\nu_1\right)}\cdot\frac{1+\nu}{E}\cdot(2\cdot\nu-1)\cdot\sum_{n=1}^{N}\alpha_n\cdot G(\xi,n)\ \ldots$$
$$+\varepsilon_l\left(E,E_1,\nu,\nu_1\right)\cdot\sigma_{xx\_remote}\left(\gamma,\lambda,\sigma_1\right)+\varepsilon_k\left(E,E_1,\nu,\nu_1\right)\cdot\sigma_{yy\_remote}\left(\gamma,\lambda,\sigma_1\right)-\alpha_p\cdot\left(f_{\Delta p}(\xi)\cdot\Delta p_{max}+p_0\right)$$

(10.9)

$$\tau_{xy\_res}(\xi) := \frac{E_1}{2\cdot\left(1+\nu_1\right)}\cdot\frac{\Delta u(\xi)}{h_{max}\cdot f_h(\xi)}-\varepsilon_d\left(E,E_1,\nu,\nu_1\right)\cdot\sum_{n=1}^{N}\beta_n\cdot G(\xi,n)+\varepsilon_m\left(E,E_1,\nu,\nu_1\right)\cdot\tau_{xy\_remote}\left(\gamma,\lambda,\sigma_1\right) \tag{10.10}$$

Generate data points along the reservoir. First, choose the number of data points along the reservoir:

$M\_res := 99$

$i\_res := 1,2\,..\,M\_res+1$

Set the limits of the reservoir:

$\xi_{min\_res} := -0.999$ $\xi_{max\_res} := 0.999$

$$\xi\_res_{i\_res} := \xi_{min\_res}+\frac{\xi_{max\_res}-\xi_{min\_res}}{M\_res}\cdot(i\_res-1)$$

10.4. Stresses Inside the Reservoir

Reference
Parameters
from Section 1:

$E = 1 \cdot 10^{10} \cdot Pa$

$\nu = 0.25$

$\nu_1 = 0.35$

$\frac{E_1}{E} = 0.3333$

$\alpha_p = 1$

$\frac{p_0}{\sigma_0} = 1$

$\frac{\sigma_1}{\sigma_0} = -1$

$\lambda = 0.75$

$\frac{p_f}{\sigma_0} = 0.3$

$\frac{2 \cdot a}{h_{max}} = 10$

$\gamma = 0 \cdot deg$

$\theta = 60 \cdot deg$

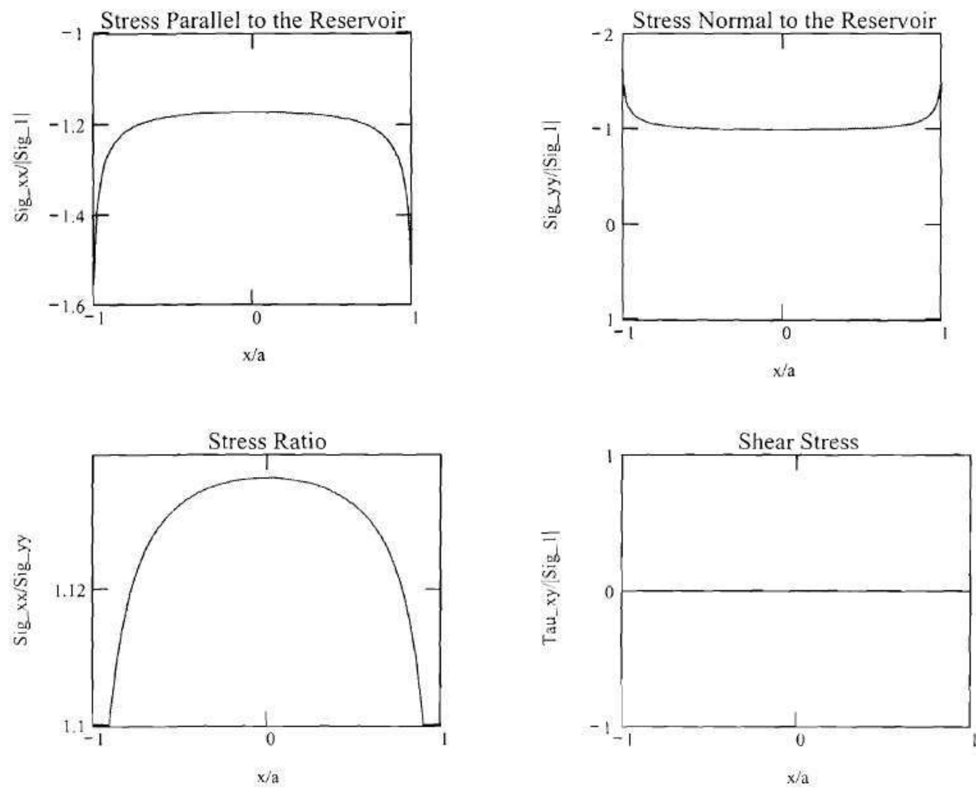


Figure 10.3.

11. References

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